Module 4:

Teaching Numbers in Base 10

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Module 4: Numbers in Base 10

Objectives

- Gain an understanding of the learning progression for numbers in base ten
- Learn math instructional strategies and activities to teach numbers in base ten

Standards

Work with numbers 11–19 to gain foundations for place value.

K.NBT.1 Compose and decompose numbers from 11 to 19 into 10 ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of 10 ones and one, two, three, four, five, six, seven, eight, or nine ones.

TEAM Alignment

- Teacher Content Knowledge
- Activities and Materials
- Questioning
- Thinking

Reflection

Think about all of the mathematical ideas we have talked about up to this point. How would you use any or all of these ideas to help your students learn about numbers in base ten?

Learning Progressions in Math

Children follow natural developmental progressions in learning and development. As a simple example, children first learn to crawl, which is followed by walking, running, skipping, and jumping with increased speed and dexterity. Similarly, they follow natural developmental progressions in learning math; they learn mathematical ideas and skills in their own way. When educators understand these developmental progressions, and sequence activities based on them, they can build mathematically enriched learning environments that are developmentally appropriate and effective.

- Retrieved from http://www.child-encyclopedia.com/numeracy/according-experts/learning-trajectories-early-mathematics-sequences-acquisition-and

The table below outlines what we want students to be able to understand and do when learning numbers in base ten.

What skills do your students need to have mastered to begin on this learning progression? Talk with your group, and, in your opinion, list the skills that a child would need to have mastered before beginning work on numbers in base ten.

Understand	Do
Concepts/Big Ideas • Quantities represented by numbers can be composed and decomposed into part-whole relationships (by place value up to 20 in K). • The base 10 number system allows for a new place-value unit by grouping ten of the previous place-value units (and this process can be iterated to obtain larger and larger place-value units). • The value of a digit in a written numeral depends on its place, or position, in a number. • Each composition or decomposition can be recorded by a drawing ***or*** equation (e.g., 18 = 10+ 8). **Models/Representations** • Discrete counters • Counters that can stack, group together (tiles, straws banded together, snap cubes, e.g.) • Rekenreks • Five frames, 10 frames • Tallies to five, 10	**K.NBT.1 Gain foundations for place value** • Represent numbers 11-19 with ones units • Associate the number 10 with a collection of ten ones (a "tens" unit) • Representing 11-19: compose ten ones (units) into a bundle of 10 to enable students to "see" a grouping of ten represented • Draw pictures representing the numbers 11-19 showing a bundle/group of ten with further units to represent the ones/units **Compose numbers 11-19 into ten ones and some further ones** • using objects • using drawings • written representation (drawing ***or*** equation) **Decompose numbers 11-19 into ten ones and some further ones:** • using objects • using drawings • written representation (drawing ***or*** equation)

- Retrieved from https://salkeiz-cia.orvsd.org/sites/salkeiz-cia.orvsd.org/files/KCC_PlaceVal_Deconst.pdf

Video Lesson Example

https://www.teachingchannel.org/videos/kindergarten-counting-cardinality-lesson

- How did the teacher make connections to the students' prior learning?

- How were the students encouraged to share and learn from each other?

- What supports does Ms. Lassiter use to further her students' thinking?

Group Activity: Plan a learning activity

Count off from one to six. Get into a group with your number. Thinking about everything we have talked about so far, plan a learning activity to teach teen numbers. Your group number's task is below.

Group 1: Demonstrate a lesson using subitizing and ten frames with teen numbers

Group 2: Demonstrate a lesson on number representations for the teen numbers

Group 3: Demonstrate a number talk for teen numbers

Group 4: Demonstrate a think aloud for decomposing a teen number

Group 5: Demonstrate a lesson teaching decomposing teen numbers with the CRA progression

Group 6: Demonstrate an addition lesson using teen numbers

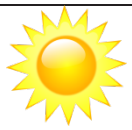
Key Idea #7

Developmental progressions refer to sequences of skills and concepts that children acquire as they build math knowledge.

- Retrieved from https://ies.ed.gov/ncee/wwc/pdf/practice_guides/early_math_pg_111313.pdf

Clear or Cloudy?

Think about the ideas we have discussed. What is clear to you? What can you use in your classroom in August? What ideas are still a little cloudy? Use the space below to list your thoughts and leave them by the door. We will talk about any "cloudy" ideas after the break.

Clear 	Cloudy

- Retrieved from http://blog.teacherspayteachers.com/5-fast-ways-close-a-lesson/

Group Activity: Teen Number Scoot

Materials needed: double ten frames, manipulatives and counting mats, base ten blocks, dry erase boards and markers (or pencil and paper), number cards for the teen numbers you are working on

Directions:

1. Put a number card on each table, along with the materials to make that number in the following ways:
    - double ten frame and counteten
    - counting manipulatives on a mat
    - with base ten blocks (rods and units)
    - writing up to that number
2. Divide students into groups. At each table they have to make the number on the double ten frame, count out that many manipulatives on the mat, use the base ten blocks to make the number, and write up to that number from one.
3. Check each table as they finish.
4. Rotate groups so all groups get to every number.

*You can start with 11-15, and then move on to 16-20 when ready.

These are resources you can print and use for the teen number scoot activity, if you don't have access to manipulatives.

Printable double ten frame
http://www.mathwire.com/templates/double10framemat.pdf

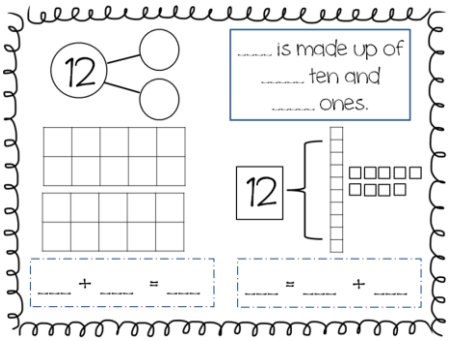
Printable base 10 blocks
http://domathtogether.com/wp-content/uploads/2013/06/Printable-Base-ten-blocks.pdf

Printable number cards
http://esgidocs.esgisoftware.com/Number+Recognition+Activity.pdf

Printable manipulatives
https://www.teachervision.com/tv/printables/scottforesman/Math_K_TTT_12.pdf

Key Idea #8

When you have students practice composing and decomposing teen numbers, you are laying the foundation for teaching place value.



Resources for Teaching Numbers in Base Ten

Virtual Manipulatives

http://nlvm.usu.edu/en/nav/category_g_1_t_1.html

Harry Kindergarten songs—YouTube

https://www.youtube.com/watch?v=1W5aYi3lkho

https://www.youtube.com/watch?v=uedvwH6Ay18

Interactive Online Games

http://www.mhschool.com/math/mathconnects/assets/mhln/00139857/00139857.swf

http://www.mhschool.com/math/mathconnects/assets/asset_view.html?s=CJ_K_07A&type=CJ

http://www.mhschool.com/math/mathconnects/assets/asset_view.html?s=SEC_K_07C&type=SEC

Literature Connection

Meet the Teens by Marcie Cooper

The teen numbers are moving in, right next to 10, 11, and 12! The book has great rhymes to help students remember the names of the teen numbers.

Chicka Chicka 1 2 3 by Bill Martin, Jr.

This book is a companion book to *Chicka Chicka Boom Boom*. The numbers are climbing up the apple tree. It names numbers up to 20, and then counts by tens to 100.

Vocabulary Connection for Numbers in Base Ten

- **tens**—a group of 10 items counted as a whole
- **ones**—items counted individually
- **rod**—base 10 block that equals 10
- **unit**—base 10 block that equals one
- **compose**—putting parts together to make a number
- **decompose**—breaking a number into parts

Can you think of other math vocabulary that students need to know?

Listening and Speaking Connection

How can you incorporate listening and speaking skills into the activities shared today? What questions can you ask students? What type of responses should you hear?

Possible Teacher Questions	Possible Student Responses
Can you show me a group of 15?	This is 10, and I have five more.
How many is 14?	It is a group of 10 and four more.
Why do I need two 10 frames to show the teen numbers?	The numbers all have a 10 and some more ones.
Can you count this group of blocks for me? (13)	Students points to the rod and says 10, then points to the units and counts on, 11, 12, 13.

What other questions could you ask?

Writing Connection: Math Journals

- After reading the book *Meet the Teens*, have students write a story about one of the teen numbers.
- Tell students to divide their journal paper into four sections and give them a teen number to represent four different ways: pictures, equation, word, tally marks, 10 frames, etc.
- Have students write a story problem using teen numbers.

Putting it all together with Instructional Tasks

Using Instructional Tasks

Research indicates that student learning is increased when teachers regularly engage students in cognitively demanding tasks. Teachers must recognize, however, that there are many factors that contribute to the overall effectiveness of using high level tasks with their students. Teachers should carefully select tasks and use them appropriately with their students. Below is a series of questions for teachers to consider when using instructional tasks in their classroom.

1. What are the mathematical goals for using this task?

- Which standards does the task target?
- Which Standards for Mathematical Practices will my students experience by working on this task?
- How does this task build upon prior knowledge?
- How will I know that students have met the mathematical goals?

2. Have I worked the task and anticipated possible solution paths that my students may take?

- Do solution paths include a variety of representations and/or strategies?
- What are common misconceptions students may have when working on the task?

3. How will I maintain the cognitive demand of the task?

- Does the task have multiple entry points to provide access for all learners?
- Does the task allow my students to explore, investigate, and make sense of mathematical ideas on their own?
- Does the task provide personal challenge and productive disequilibrium for all students?
- Am I using Accountable Talk to assure that all students' voices are heard and are actively engaged in the task?
- How will I respond when students struggle or have difficulty starting the task?
- How can I advance the thinking of students who finish the task early?
- Are students required to justify and defend their solutions?
- What questions will I ask students to assess their learning and advance them toward the mathematical goals?

4. How can I summarize the learning during the whole group discussion?

- How will I select and sequence the solution paths to leverage and advance student thinking about the mathematical goal(s) of the lesson?
- How will I help students make connections among students' solution paths and mathematical goal(s)?

Teachers using high-level instructional tasks in their classrooms should follow the launch–explore -summarize lesson structure. It is recommended that teachers give students a regular diet of high level tasks, while also recognizing that there are other productive mathematical activities to use with students.

K.CC.B.4 *The Napping House* Task

This particular task helps illustrate Mathematical Practice Standard 1, Make sense of problems and persevere in solving them. Kindergartners are exposed to multiple problems through the story, *The Napping House*. As students listen to the story they use counters on a ten frame to keep track of each additional person/animal who gets in the bed until the flea bites the mouse. Then the story changes to subtraction as people/animals start to leave the bed. Throughout this guided task, students are introduced to the processes of problem-solving in a non-threatening way. They are able to unpack the parameters of the problem by manipulating the counters one at a time. This allows them to make sense of the actions occurring in the story. These concrete objects help them to conceptualize and solve each problem as posed in the story. The teacher can guide this conceptualization by stopping after each action and asking questions such as,
"What just happened in the story?" "How are we going to show that on our ten frames?" "How many are in the bed now?" and "How do you know how many are in the bed?"

Materials Needed

- ✓ *The Napping House* by Audrey Wood
- ✓ One ten-frame for each child
- ✓ 6-10 counters per child

Directions

- ✓ The teacher reads *The Napping House* to the class, stopping each time a person or animal gets into the bed so the students can add a counter to the ten-frame.

- ✓ After each page, stop to ask the children how many are sleeping in the bed after each counter is added to the ten-frame. For example, at the beginning there should be 1 counter for granny. When the child gets in the bed, there should be 2 counters.

- ✓ Have the children tell how many people there are in the bed now. Do this after each counter is added to the ten-frame. There should be 6 counters on the ten-frame once the wakeful flea is added.

- ✓ Once the flea bites the mouse, the children should begin taking the counters off the ten-frame to represent how many people/animals are still in the bed. For example, once the flea bites the mouse, there are only 4 people/animals left in the bed.

- ✓ By the end of the story, there should be no counters on the ten-frame.

Task Purpose

The purpose of the task is for students to use the context of *The Napping House* to connect counting and cardinality. The teacher or students could also write a simple equation on the classroom dry-erase board each time another person or animal gets into the bed. This would connect counting to addition and subtraction for the students, and would connect with standard K.OA.1. For example, when the child gets in with granny, the equation would be 1+1=2 and so on for each animal added to the bed.

Students can be given 6 counters or 10 counters, depending on their ability level. Students who need more scaffolding should be given 6 counters so that they cannot accidentally miscount and place too many counters on the ten-frame. However, students who are proficient counters should be given 10 counters. This will make them responsible for accurately keeping the count.

Retrieved from https://www.illustrativemathematics.org/content-standards/tasks/1149

K.CC More and Less Handfuls Task

Materials Needed

- ✓ A variety of manipulatives for counting
- ✓ Student recording sheet (see setup)

Setup

- ✓ On a sheet of plain paper write the following sentence frame at the bottom:

I have ____counters. I have _____ (more than/less than/ the same as) my partner. My partner has_____ counters.

- ✓ Copy one sheet per student.
- ✓ Set out a variety of math manipulatives at each table group.
- ✓ Have students work in pairs.

Directions

- ✓ Each student grabs two handfuls of counters.
- ✓ The student combines his/her handfuls into one collection and then counts them.
- ✓ The student then draws and records the quantity on a student-recording sheet.
- ✓ Student partners then complete the sentence frame at the bottom of the page together, stating how many each person had and if they have more or less than their partner.
- ✓ If students are having a hard time with the quantity generated from two handfuls have them only grab one handful of counters to start with.
- ✓ Watch the student's counting strategies. Students should have an organized method for keeping track of items they have already counted. They may use a "pull-off" strategy, moving items from one side of the table to the other, or line them up in a straight line. If students are struggling give them a large paper plate and have them count the items by moving them onto the plate and check it by moving them off the plate.
- ✓ Have the students' progress to grabbing one handful each of two different types of counters in order to ensure they can combine unlike groups into a single collection.

Solution

- ✓ Students should have a pictorial representation of the collection of counters as well as a completed sentence frame such as; "I have 9 counters. I have more than my partner. My partner has 7 counters."
- ✓ Begin with larger types of manipulative such as counting bears, cubes, etc. so that the handfuls do not contain as many items. Have students grab two handfuls from the same type of counters and then vary it so that they grab one handful each from two different types of counters. Progress to smaller counters like buttons or two color counters to increase the quantity that they are dealing with because students will be able to grab larger amounts.

Math Task Resources

https://www.illustrativemathematics.org/content-standards/K

http://commoncoretasks.ncdpi.wikispaces.net/K+Tasks

http://www.insidemathematics.org/common-core-resources/mathematical-content-standards/standards-by-grade/kindergarten

Task Analysis Guide

Lower-Level Demands	***Higher-Level Demands***
Memorization Tasks • Involves either producing previously learned facts, rules, formulae, or definitions, OR committing facts, rules, formulae or, definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	Procedures With Connections Tasks • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little	***Doing Mathematics Tasks*** • Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).

ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	• Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demands self-monitoring or self-regulation of one's own cognitive processes. • Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Mathematics Teaching in the Middle School. Also in: Stein, Smith, Henningsen, & Silver (2000). Implementing standards-based mathematics instruction: A casebook for professional development, p. 16. New York: Teachers College Press.

Kindergarten Instructional Tasks

All instructional materials previously available on TNCore.org are now hosted at eduToolbox.org. Kindergarten resources can be found here: http://edutoolbox.org/tntools/list/grade/819/955/0#958

- **The Seashell Collections** (K.CC.B.4, K.CC.B.4a, K.CC.B.4b, K.CC.B.4c, K.CC.B.5, K.CC.C.6)
- **The Four-Leaf Clover** (K.OA.A.1, K.OA.A.2, K.OA.A.4)
- **Counting Balls** (K.CC.B.4, K.CC.B.4a, K.CC.B.4b, K.CC.B.4c, K.CC.B.5)
- **Sammy's Pets** (K.OA.A.1, K.OA.A.3)

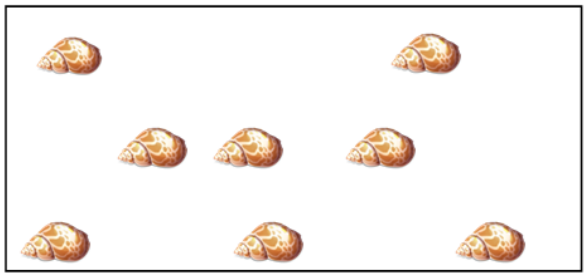
Task: The Seashell Collections **Kindergarten**

Carrie, Ashley, and Suzanne are collecting seashells. Each collection is shown below.

Carrie's Seashell Collection

Ashley's Seashell Collection

Suzanne's Seashell Collection



Count the number of shells in each girl's collection. How many shells are in each collection?

Suzanne thinks she has the most seashells. Do you agree or disagree with Suzanne? Explain your answer.

Teacher Notes:

A student page is provided so that students may have easy access to count the number of shells in each collection. The seashell collections contain the same number of shells but are arranged in a line, a rectangular array, and a scattered configuration. Students should understand that the arrangement of the objects in a set does not affect the number of objects in the set. Students should also realize that another important part of counting accurately is being able to keep track of what has already been counted and what remains to be counted.

If students should have trouble understanding that all of the seashell collections contain the same number of shells, you might suggest that they cover the seashells with counters, using a different color for each girl. Students can then determine that the number of objects is the same by matching the counters with a one-to-one correspondence or rearranging the counters to make 3 identical sets with the same arrangement.

Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality. **K.CC.B.4a** When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. **K.CC.4b** Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. **K.CC.4c** Understand that each successive number name refers to a quantity that is one larger. **K.CC.B.5** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. **K.CC.C.6** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1] [1] Include groups with up to ten objects.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings	
• Counting tells how many things are in a set. When counting a set of objects, the last word in the counting sequence names the quantity for that set. (Van de Walle) • Counting includes one-to-one correspondence, regardless of the kind of objects in the set and the order in which they are counted. (NCTM) • The relationship between one quantity and another can be an equality or inequality relation. (NCTM)	
Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
Student accurately counts the number of shells in each seashell collection and determines that each collection has eight shells. Student recognizes that the number of shells in each collection is equal and Suzanne is not correct that she has the most seashells.	Assessing Questions • How did you count the number of shells in each collection? • How do you know there are eight seashells in each collection? • How do you know the number of seashells in each collection is the same? Advancing Questions • How is it possible that each collection has eight shells when the collections look so different? • What if each collection contained different kinds of shells, would they still have the same number?
Student counts the number of shells in each collection. Student matches the number of shells in each collection using one-to-one correspondence to determine that the number of shells in each collection is the same. Student then determines that Suzanne is not correct that she has the most seashells.	Assessing Questions • How did you count the number of shells in each collection? • How many seashells are in each collection? Advancing Questions • How is it possible that each collection contains the same number of shells when the collections look so different? • If each set contain eight shells, are they equal? How do you know?
Possible Student Misconceptions	
Student does not accurately count the number of shells in each collection.	• How did you determine the number of shells in each collection? • Count the shells again. Did you get the same number?
Student does not understand that the number of shells in each collection is the same. Student thinks that Suzanne's collection contains more because the collection is scattered and covers a greater area.	• How many shells are in each collection? • How does the number of shells in the collection help us determine who has the most shells? • Does the arrangement of the shells affect the number of shells in the collection? Why or why not?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	Assessing Questions • How can you determine the number of shells in each collection? • What number begins the counting sequence?
If students finish early….	Assessing Questions • How many more shells does each girl need to have a total of 10 shells in her collection? • How many shells do the girls have altogether?

Discuss/Analyze
Whole Group Questions
• How did you determine the number of seashells in each collection? • Did the arrangement of the seashells make a difference or change the number of shells in each collection? • How did you determine whether or not Suzanne had the most shells?

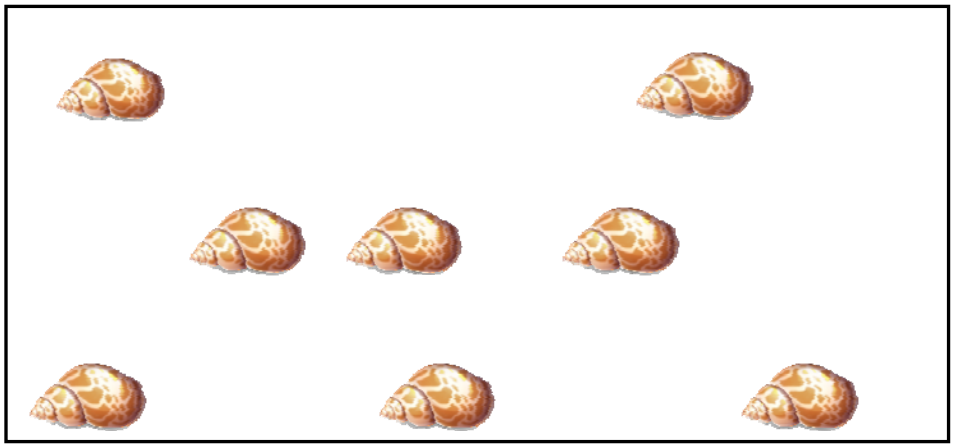
The Seashell Collections Task

Carrie, Ashley, and Suzanne are collecting seashells. Each collection is shown below.

Carrie's Seashell Collection

Ashley's Seashell Collection

Suzanne's Seashell Collection



Count the number of shells in each girl's collection. How many shells are in each collection?

Suzanne thinks she has the most seashells. Do you agree or disagree with Suzanne? Explain your answer.

Task: The Four-Leaf Clover Task **Kindergarten**

Max is collecting four-leaf clovers. He found 5 clovers while playing at the park. He found 2 clovers in his yard. How many four-leaf clovers does Max have? Use a drawing, objects, or a number sentence (equation) to show how you found your answer.

Max claims that if he finds 3 more clovers he will have 10. Do you agree or disagree with Max? Use a drawing or a number sentence (equation) in your explanation.

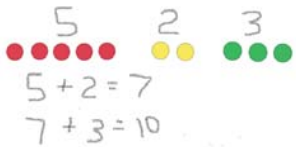
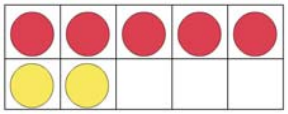
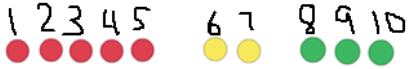
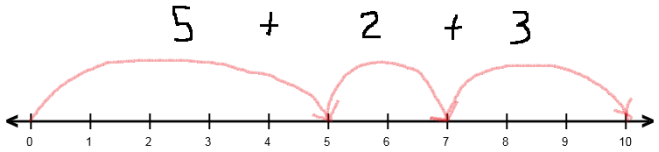
Teacher Notes:

Cubes, counters, or other manipulatives should be available for students to use as needed. A part-part-whole map or a tens frame may be helpful for some students to visualize and to make sense of the problem. The term "number sentence" is used instead of "equation". Teachers should model the term "equation" but students may continue to use the term "number sentence". Students may choose not to write an equation, but should be able to explain how they found the answer with a drawing or model. If students do not write an equation, the teacher may choose to model this in the whole group discussion.

Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. **K.OA.A.2** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. **K.OA.A.4** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. [1] Drawings need not show details, but should show the mathematics in the problem.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings

- The addition of whole numbers is based on sequential counting.
- Addition equations can be used to describe situations that involve combining quantities.

Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
Direct Modeling with manipulatives or drawings Student draws or counts 5 objects and 2 objects and recognizes that the quantities should be combined to get a total of 7.  Student then determines that 7 and 3 more will equal 10. Student agrees with Max's statement that he needs 3 more to have a total of 10. Using a tens frame, student can count that 5 and 2 are 7.  Student agrees that if Max had three more he would have a total of ten.	**Assessing Questions** • How did you determine the number of clovers that Max has? • How did you determine the number of clovers that Max will need to have 10 clovers? • Describe your drawing or equation. How did this help you determine your solution? • What do the 5 and 2 represent in your equation? **Advancing Questions** • You have determined that Max has 7 clovers. How can you determine how many more he needs to have 10? • Can you write an equation to represent this situation? (for students who did not write an equation)
Counting On Students begins with 5 (or counts up to 5) and counts on 2 more for a total of 7 and then counts on 3 more for a total of 10. Student then agrees with Max's statement that if he had 3 more he would have a total of 10.  This could be illustrated with a number line in one of the following ways:  	**Assessing Questions** • How did you determine the number of clovers that Max has? • How did you determine the number of clovers that Max will need to have 10 clovers? • Where did you start counting? How do you know where to stop counting? • What do the 5 and 2 represent in your equation? **Advancing Questions** • You have determined that Max has 7 clovers. How can you determine how many more he needs to have 10? • Can you write an equation to represent this situation? (for students who did not write an equation) • How can a number line be used to model this situation?

Possible Student Misconceptions	
Student does not correctly add the number of clovers and finds an incorrect sum.	• How do you know where to start counting? • What does the starting number represent? • How much will you add to the starting number? • What do the numbers 5, 2 and 3 represent? • How can counting help you find your answer?
Student does not agree that if Max finds 3 more clovers then he will have 10.	• How many clovers does Max have after he finds 5 and 2 clovers? • If Max has 7 clovers, how many more will he need to have 10?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• Describe the story problem. • How many clovers does Max have after he visits the park and his yard? • How can you model this story problem with cubes?
If students finish early….	• How many more clovers will Max need to find so that he has 12 clovers? • If Max finds 5 clovers and 2 clovers, but then loses 1 clover. How many more will he need to have 10 clovers?
Discuss/Analyze	
Whole Group Questions	
• Describe how you found the total number of clovers that Max found at the park and in his yard? • How can a tens frame, cubes or a number line help us model this problem? • How can we represent this problem with an equation?	

The Four-Leaf Clover Task

Max is collecting four-leaf clovers. He found 5 clovers while playing at the park. He found 2 clovers in his yard. How many four-leaf clovers does Max have? Use a drawing, objects and or a number sentence to show how you found your answer.

Max claims that if he finds 3 more clovers he will have 10. Do you agree or disagree with Max? Use a drawing or a number sentence in your explanation.

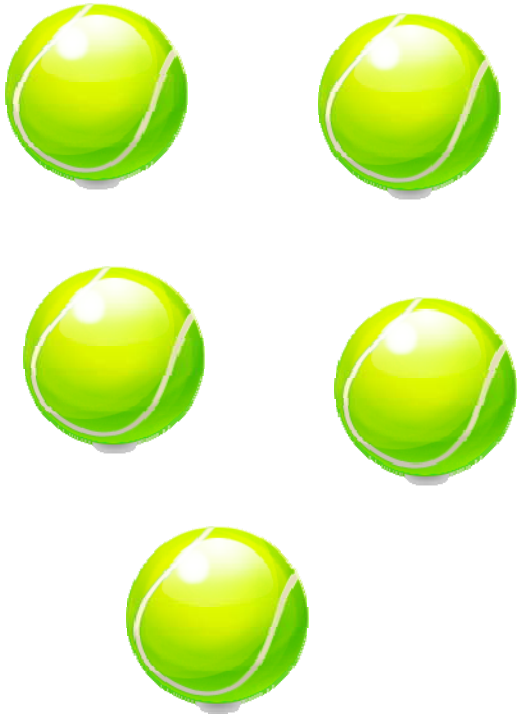
Task: Counting Balls	**Kindergarten**
Zanna and Sage like to play with different kinds of sports balls. Zanna has 5 tennis balls and Sage has 5 golf balls. Zanna says that she has more balls because the tennis balls are bigger. Do you think Zanna is right? Explain your answer with words and pictures. (See e below for pictorial representation of the size of the balls)	
Teacher Notes:	
Use of actual tennis balls and golf balls or cut outs of the attached figures as manipulatives is recommended. Conservation of number is essential to conceptual understanding of counting.	
Common Core State Standards for Mathematical Content	**Common Core State Standards for Mathematical Practice**
Count to tell the number of objects **K.CC.B.4** Understand the relationship between numbers and quantities, connect counting to cardinality. **K.CC.B.4a** When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. **K.CC.B.4b** Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. **K.CC.B.4c** Understand that each successive number name refers to a quantity that is one larger. **K.CC.B.5** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle or as many as 10 things in a scattered configuration; given a numb3er from 1-20 count out that many objects.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Essential Understandings	
• Counting includes one-to-one correspondence, regardless of the kind of objects in the set and the order in which they are counted. • When counting objects in a group/set, the last number stated names the total number of objects in that group/set.	
Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
Student will count each set of balls and label or name them with the correct number. Student will then state that because there is the same number of tennis balls that there is of golf balls Zanna and Sage have the same or equal groups so Zanna's claim is incorrect.	Assessing Question: How did you decide that Zanna was not right? What strategy did you use to determine whether Zanna had more? Advancing Question: What would you change in the problem to make Zanna's claim correct? Does the size of the objects make a difference in this problem? Is there another way to show /explain your answer?

Student may line the balls up using one-to-one correspondence to show that the groups/sets of tennis balls and golf balls are the same. Zanna's claim is incorrect.	<u>Assessing Question:</u> How did you know to line the balls up? Did that make it easier? Why or why not? <u>Advancing Question:</u> What would you change in the problem to make Zanna's claim correct? Does the size of the objects make a difference? Is there another way to show /explain your answer?
Student may use the pictorial representation to connect one tennis ball to one golf ball until all are paired and relate that the girls have the same number.	<u>Assessing Question:</u> Tell me how you decided to draw the lines to match a tennis ball and a golf ball. Tell me how matching a tennis ball with a golf ball helped you answer the question. <u>Advancing Question:</u> If each girl had some of the tennis balls and some of the golf balls could you use the same strategy? Explain your answer.
Possible Student Misconceptions	
Students may think that there are more tennis balls because they are larger than the golf balls. When placed in a line, the line of tennis balls is longer than the line of golf balls so it may appear there are more. Students may count the balls in random order and count some of them more than once.	<u>Assessing Questions:</u> Why do you think Zanna said she had more? How could you find out? <u>Advancing Question:</u> Is there a way to arrange the balls that would help you? If you move the balls around would that change your answer?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	<u>Assessing Question:</u> Why do you think Zanna said she had more balls? What can you do to see how many balls each girl has? <u>Advancing Question:</u> Is there a way to arrange the balls that would help you see if Zanna is correct?
If students finish early….	Have the students show how each girl could have an equal number of balls but a combination of both tennis balls and golf balls. Write number sentences to demonstrate their answers. If you gave each girl two more balls would they still have an equal number of balls? Write number sentences to demonstrate their answers.

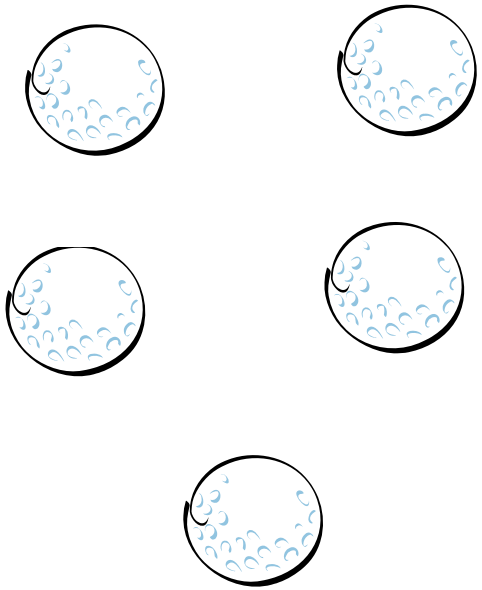
Discuss/Analyze
Whole Group Questions
• What ways did you use to count the balls? • How were the counting strategies that we shared similar and different? • How did you know if you had already counted a ball? How did you keep from counting it more than one time? • Does it matter where you start when you are counting objects? Explain your answer. • When you say the number of the last ball in a group, what does that number tell you? • If you rearrange the balls in a group and count them again, will you get the same number? Explain your answer. • If you are counting things in a group, does it matter if the things are different sizes? Explain your answer.

Tennis Balls and Golf Balls

Zanna's Tennis Balls



Sage's Golf Balls

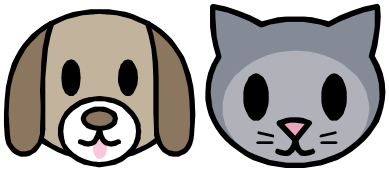


Task: Sammy's Pets	Kindergarten
Sammy has 7 pets. Some are dogs and some are cats. How many dogs and how many cats could Sammy have? Use a drawing and a number sentence (equation) to explain your answer. Choose another way to show how Sammy could have 7 pets if some are dogs and some are cats. Use a drawing and a number sentence (equation) to explain your answer.	
Teacher Notes:	
Cubes or other manipulatives should be available for students to use if needed. *Do not give students 7 cubes.* Have cubes available and allow students to count the number of cubes needed. A part-part-whole map may be helpful for some students to recognize that 7 cubes should be decomposed into two groups and that the two groups have a total of 7 pets. The term "number sentence" is used instead of "equation". Teachers may choose to model the term "equation" but students may choose to continue to use the term "number sentence".	
Common Core State Standards for Mathematical Content	**Common Core State Standards for Mathematical Practice**
K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. **K.OA.A.3** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). [2] Drawings need not show details, but should show the mathematics in the problem.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Essential Understandings	
• A quantity can be decomposed into two parts in more than one way. • A quantity can be decomposed into two parts and the whole quantity remains the same. The decomposition may be recorded as an addition equation.	
Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
Direct modeling with manipulatives: Student counts 7 objects and divides the objects into two groups – one to represent dogs and one to represent cats. (A part-part-	Assessing Questions • Which group of cubes represents the number of dogs and which represents the number of cats? • Why did you start with 7 cubes?

whole map may be helpful for some students.)	• Describe how you found the answer to the problem. Advancing Questions • What are other solutions to this problem? • Is it possible for Sammy to have the same number of cats and dogs? Why or why not?
Counting on from a number less than 7: Student chooses a number less than 7 to represent the number of dogs and then counts on to determine the number of cats needed to make a total of seven pets. Possible representations may include objects or number lines.	Assessing Questions • Describe how you found the answer to the problem. • Why did you start with a number less than 7? Advancing Questions • What are other solutions to this problem? • Is it possible for Sammy to have the same number of cats and dogs? Why or why not?
Known Addition Combinations Student chooses a known combination for seven and recognizes that each addend could represent the number of cats or dogs. Examples could include: 1 + 6 = 7 so Sammy has 1 dog and 6 cats 1 + 6 = 7 so Sammy has 1 cat and 6 dogs 2 + 5 = 7 so Sammy has 2 dogs and 5 cats	Assessing Questions • Describe how you found the answer to the problem. • I notice that you said Sammy could have 1 dog and 6 cats or 6 cats and 1 dog. These use the same numbers. Describe how they are different. Advancing Questions • What are other solutions to this problem? • Is it possible for Sammy to have the same number of cats and dogs? Why or why not?
Possible Student Misconceptions	
Student inaccurately counts the number of cubes in each set or inaccurately counts when adding up to 7.	Do the number of dogs and cats equal 7? How do you know? Do you think it is possible for Sammy to have 8 dogs? Why or why not?
Student decomposes the 7 into more than two parts.	What does each number represent? How could we represent dogs and cats with the cubes? (Perhaps use two colors, etc.)
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	Tell me what you know about Sammy's pets. Show me with cubes the number of pets that Sammy has.
If students finish early….	What is greatest number of dogs or cats that Sammy could have if Sammy has both cats and dogs? How can the equation 5 + 2 = 7 represent two different solutions? What are all of the possible solutions to this problem and how do you know you have found all solutions?

Discuss/Analyze
Whole Group Questions
How can the seven be decomposed into two groups in more than one way? Describe a number sentence (equation) that could be used to show how 7 can be decomposed into 2 parts. Describe the relationship between the number sentence and number of pets Sammy has.

Sammy's Pets Task



Sammy has 7 pets. Some are dogs and some are cats.

How many dogs and how many cats could Sammy have?
Use a drawing and a number sentence to explain your answer.

Choose another way to show how Sammy could have 7 pets if some are dogs and some are cats?
Use a drawing and a number sentence to explain your answer.

Group Project

After reading the task examples, work with your grade level groups to create a Math task for your students.

- ✓ What math practice standards can you address?
- ✓ How can you connect the task to literacy components?
- ✓ What writing skills can you incorporate to the task?

Math Task Template

PART 1: SELECTING AND SETTING UP A MATHEMATICAL TASK	
What are your mathematical goals for the lesson? (i.e., what do you want students to know and understand about mathematics as a result of this lesson?)	
• What are your expectations for students as they work on and complete this task? • What resources or tools will students have to use in their work that will give them entry into, and help them reason through, the task? • How will the students work—independently, in small groups, or in pairs—to explore this task? • How will students record and report their work?	
How will you introduce students to the activity so as to provide access to *all* students while maintaining the cognitive demands of the task?	

PART 2: SUPPORTING STUDENTS' EXPLORATION OF THE TASK	
As students work independently or in small groups, what questions will you ask to— • help a group get started or make progress on the task? • focus students' thinking on the key mathematical ideas in the task? • assess students' understanding of key mathematical ideas, problem- solving strategies, or the representations? • advance students' understanding of the mathematical ideas?	
• How will you ensure that students remain engaged in the task? • What assistance will you give or what questions will you ask a student (or group) who becomes quickly frustrated and requests more direction and guidance is solving the task? • What will you do if a student (or group) finishes the task almost immediately? How will you extend the task so as to provide additional challenge?	

PART 3: SHARING AND DISCUSSING THE TASK	
• How will you orchestrate the class discussion so that you accomplish your mathematical goals? • Which solution paths do you want to have shared during the class discussion? In what order will the solutions be presented? Why? • What specific questions will you ask so that students will— 1. make sense of the mathematical ideas that you want them to learn? 2. expand on, debate, and question the solutions being shared? 3. make connections among the different strategies that are presented? 4. look for patterns? 5. begin to form generalizations? • What will you see or hear that lets you know that *all* students in the class understand the mathematical ideas that you intended for them to learn?	

Adapted from: Smith, Margaret Schwan, Victoria Bill, and Elizabeth K. Hughes. "Thinking Through a Lesson Protocol: Successfully Implementing High-Level Tasks." *Mathematics Teaching in the Middle School 14* (October 2008): 132-138.

Reflecting on the Training

Thinking about the topics that have been shared, what will you take back to your school and share with your teammates? List your ideas below.

- **Numeracy-Rich Classroom Components**

- **Subitizing and Number Representations**

- **Math Talk (Number Talks and Think Alouds)**

- **Five and 10 Frames**

- **Addition and Subtraction**

- **Numbers in Base 10**

- **Math Tasks**