

# Tennessee Mathematics Standards

Approved July 30, 2010

## Pre-Calculus #3126

### Standard 1 – Mathematical Processes

#### Course Level Expectations

CLE 3126.1.1 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely in mathematical reasoning.

CLE 3126.1.2 Apply and adapt a variety of appropriate strategies to problem solving, including testing cases, estimation, and then checking induced errors and the reasonableness of the solution.

CLE 3126.1.3 Develop inductive and deductive reasoning to independently make and evaluate mathematical arguments and construct appropriate proofs; include various types of reasoning, logic, and intuition.

CLE 3126.1.4 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic), to solve problems, to model mathematical ideas, and to communicate solution strategies.

CLE 3126.1.5 Recognize and use mathematical ideas and processes that arise in different settings, with an emphasis on formulating a problem in mathematical terms, interpreting the solutions, mathematical ideas, and communication of solution strategies.

CLE 3126.1.6 Employ reading and writing to recognize the major themes of mathematical processes, the historical development of mathematics, and the connections between mathematics and the real world.

CLE 3126.1.7 Use technologies appropriately to develop understanding of abstract mathematical ideas, to facilitate problem solving, and to produce accurate and reliable models.

#### Check for Understanding (Formative/Summative Assessment)

- ✓ 3126.1.1 Give a sequence of algebraic or mathematical reasons to justify the validity of the steps in a mathematical proof.
- ✓ 3126.1.2 Use algebraic properties to develop a valid sequence of mathematical statements.
- ✓ 3126.1.3 Correctly use summation notation; expand and collect expressions in both finite and infinite settings.
- ✓ 3126.1.4 Derive and apply the formulas for the area of sector of a circle.

- ✓ 3126.1.5 Conduct simple experiments or investigations to collect non-linear data to answer questions of interest.
- ✓ 3126.1.6 Understand the different representations of a function; discuss the criteria (such as the type of function and the problem under consideration) for determining which representation is most helpful.
- ✓ 3126.1.7 Analyze situations, develop mathematical models, or solve problems using linear, polynomial, trigonometric, exponential, or logarithmic equations or inequalities symbolically or graphically.
- ✓ 3126.1.8 Draw qualitative graphs (sketches) of functions (linear, quadratic, cubic, square root, absolute value, reciprocal, trigonometric, exponential, logarithmic, and greatest integer) and describe their general shape/trend.
- ✓ 3126.1.9 Demonstrate diagrammatically composition of functions and inverse of functions; discuss similarity to and differences from arithmetic operations of functions.
- ✓ 3126.1.10 Make inferences or predictions using an algebraic model of a situation.
- ✓ 3126.1.11 Discuss interpolation vs. extrapolation and the validity of the resulting estimates.
- ✓ 3126.1.12 Discuss the changes in mathematics that arose through the development of function notation, Cartesian coordinates, base  $e$ , and other mathematical ideas discussed in pre-calculus.
- ✓ 3126.1.13 Establish accurate and consistent use of units in the presentation of answers to applied questions.
- ✓ 3126.1.14 Use graphing calculators and computer spreadsheets to analyze qualities of a function

## **Standard 2 – Number & Operations**

### **Course Level Expectations**

- CLE 3126.2.1 Understand the capabilities and the limitations of calculators and computers in solving problems.
- CLE 3126.2.2 Represent, interpret or compare expressions for real numbers, including expressions utilizing exponents and logarithms.
- CLE 3126.2.3 Develop the ability to recognize the difference between algebraic and transcendental expressions; be able to classify a number (Natural, integer, rational, etc.) written in a complex format.
- CLE 3126.2.4 Develop facility with simplification of complex algebraic expressions involving exponential notation, logarithmic notation, rational notation, and radicals
- CLE 3126.2.5 Be able to calculate vector arithmetic and vector length.
- CLE 3126.2.6 Recognize vectors as elements (i.e. numbers) that have their own form of arithmetic operations in their own system of elements.
- CLE 3126.2.7 Recognize series as an identification of a number which can be identified as a specific numeral or only approximated.

### **Check for Understanding (Formative/Summative Assessment)**

- ✓ 3126.2.1 Use calculators appropriately and make estimations without a calculator regularly to detect potential errors.
- ✓ 3126.2.2 Demonstrate round-off error, over-flow error, and errors in mode settings (ex. Degree vs. radians) with particular examples.
- ✓ 3126.2.3 Compare exponential and logarithmic expressions.
- ✓ 3126.2.4 Recognize the difference between continuous and discrete situations.
- ✓ 3126.2.5 Classify real numbers and order real numbers that include transcendental expressions, including roots and fractions of  $\pi$  and  $e$ .
- ✓ 3126.2.6 Simplify complex radical and rational expressions; discuss and display understanding that rational numbers are dense in the real numbers and the integers are not.
- ✓ 3126.2.7 Multiply a vector by a scalar both algebraically and graphically.
- ✓ 3126.2.8 Add vectors both algebraically and graphically.
- ✓ 3126.2.9 Calculate magnitude and direction of a vector.
- ✓ 3126.2.10 Calculate and interpret the dot product of two vectors.
- ✓ 3126.2.11 Understand that vectors are determined by the coordinates of their initial and terminal points, or by their components.
- ✓ 3126.2.12 Use vectors to model velocity and direction to solve problems.
- ✓ 3126.2.13 Determine whether a given arithmetic or geometric series converges or diverges.
- ✓ 3126.2.14 Demonstrate an understanding of sequences by representing them recursively and explicitly.
- ✓ 3126.2.15 Use Sigma notation to represent a series.
- ✓ 3126.2.16 Find the sum of a given geometric series (both infinite and finite).
- ✓ 3126.2.17 Find the sum of a finite arithmetic series.
- ✓ 3126.2.18 Use the laws of exponents and logarithms to expand or collect terms in expressions; simplify expressions or modify them in order to analyze them or compare them.

## **Standard 3 – Algebra**

### **Course Level Expectations**

- CLE 3126.3.1 Develop an understanding of functions as elements that can be operated upon to get new functions: addition, subtraction, multiplication, division, and composition of functions.
- CLE 3126.3.2 Understand how the algebraic properties of an equation transform the geometric properties of its graph.
- CLE 3126.3.3 Analyze the graph of a function, given either a sketch or a symbolic description.
- CLE 3126.3.4 Identify or analyze the distinguishing properties of exponential, polynomial, logarithmic, trigonometric, and rational functions from tables, graphs, and equations.

- CLE 3126.3.5 Apply appropriate techniques to analyze mathematical models and functions constructed from verbal information; interpret the solution obtained in written form using appropriate units of measurement.
- CLE 3126.3.6 Solve maximum/minimum value problems by converting the given verbal information into an appropriate mathematical model and analyzing the graph of that model graphically to answer the questions. Recognize the approximation necessary when solving graphically.
- CLE 3126.3.7 Solve nonlinear inequalities (quadratic, trigonometric, conic, exponential, and logarithmic).
- CLE 3126.3.8 Understand the properties of conic sections (whether displayed in equation or graphical form) and apply conic sections to model real-world phenomena.
- CLE 3126.3.9 Simulate motion using parametric equations.
- CLE 3126.3.10 Derive and use the formulas for the general term and summation of finite or infinite arithmetic and geometric series, if they exist.
- CLE 3126.3.11 Develop the concept of a limit by examining sequences and series.

**Check for Understanding (Formative/Summative Assessment)**

- ✓ 3126.3.1 Calculate the inverse of a function with respect to each of the functional operations; in other words, the additive inverse, the multiplicative inverse, and the inverse with respect to composition.
- ✓ 3126.3.2 Recognize the role that domain of function plays in the combination of functions by composition of functions.
- ✓ 3126.3.3 Identify whether a function has an inverse with respect to composition and when functions are inverses of each other with respect to composition.
- ✓ 3126.3.4 Explain why the graph of a function and its inverse are reflections of one another over the line  $y = x$ .
- ✓ 3126.3.5 Explain the relationship between the real zeros and the  $x$ -intercept of the graph of a function (polynomial, rational, exponential, logarithmic, and trigonometric).
- ✓ 3126.3.6 Identify the real zeros of the graph of a function (polynomial, rational, exponential, logarithmic, and trigonometric) in equation or graphical form.
- ✓ 3126.3.7 Identify characteristics of graphs based on a set of conditions or on a general equation such as  $y = ax^2 + c$ .
- ✓ 3126.3.8 Given a function, describe the transformation of the graph resulting from the manipulation of the algebraic properties of the equation (i.e., translations, stretches, and changes in periodicity and amplitude)
- ✓ 3126.3.9 Determine the asymptotes and end behaviors of functions.
- ✓ 3126.3.10 Determine whether a function is even, odd, or neither.
- ✓ 3126.3.11 Prove basic properties of a logarithm using properties of its inverse and apply those properties to solve problems.
- ✓ 3126.3.12 Find the inverse of an exponential or a logarithmic function.

- ✓ 3126.3.13 Visually locate critical points on the graphs of polynomial functions and determine if each critical point is a minimum, a maximum, or point of inflection.
- ✓ 3126.3.14 For a given sketch of a graph of a function, describe the concavity and locate maximums, minimums, increasing and decreasing intervals, and zeroes.
- ✓ 3126.3.15 Sketch the graph of a given a rational function and locate vertical, horizontal, and slant asymptotes, and holes in the graph if they exist.
- ✓ 3126.3.16 Solve real world problems that can be modeled using quadratic, exponential, or logarithmic functions (by hand and with appropriate technology).
- ✓ 3126.3.17 Solve nonlinear inequalities by graphing (solutions in interval notation if one-variable) by hand and with appropriate technology.
- ✓ 3126.3.18 Solve systems of nonlinear inequalities by graphing.
- ✓ 3126.3.19 Graph ellipses and hyperbolas and demonstrate understanding of the relationship between their standard algebraic form and the graphical characteristics.
- ✓ 3126.3.20 Graph circles and demonstrate an understanding of the relationship between their standard algebraic form and the graphical characteristics.
- ✓ 3126.3.21 From an equation in standard form, graph the appropriate conic section.
- ✓ 3126.3.22 Graph curves parametrically (by hand and with appropriate technology).
- ✓ 3126.3.23 Eliminate parameters by rewriting parametric equations as a single equation.
- ✓ 3126.3.24 Understand the series represent the approximation of a number when truncated; estimate truncation error in specific examples.
- ✓ 3126.3.25 Understand that lengths of curves and areas of curved regions can be defined using the informal notion of limit.
- ✓ 3126.3.26 Construct the difference quotient for a given function and simplify the resulting expression.

## **Standard 4 – Geometry & Measurement**

### **Course Level Expectations:**

- CLE 3126.4.1 Understand basic right triangle trigonometry and use it to solve problems.
- CLE 3126.4.2 Know how the trigonometric functions can be extended to the periodic functions on the real number line, derive basic formulas of these functions, and use these functions and formulas to solve problems.
- CLE 3126.4.3 Solve trigonometric equations and inequalities algebraically and graphically, by hand and with appropriate technology.
- CLE 3126.4.4 Apply trigonometric identities to rewrite expressions and solve equations.
- CLE 3126.4.5 Apply vectors to solve real world problems.
- CLE 3126.4.6 Represent situations and solve problems involving polar

coordinates.

CLE 3126.4.7 Interpret transformations of trigonometric functions.

CLE 3126.4.8 Understand the geometric interpretation of vectors and their use in real life analysis of problems.

CLE 3126.4.9 Develop an understanding of the graphic representation of vectors and vector arithmetic.

### **Check for Understanding (Formative/Summative Assessment)**

- ✓ 3126.4.1 Solve problems using the fact that trigonometric ratios (sine, cosine, and tangent) stay constant in similar triangles.
- ✓ 3126.4.2 Use the definitions of the six trigonometric ratios as ratios of sides in a right triangle to solve problems about lengths of sides and measures of angles.
- ✓ 3126.4.3 Match a trigonometric equation with its graph.
- ✓ 3126.4.4 Know that the six trigonometric functions can be extended to periodic functions on the real number line.
- ✓ 3126.4.5 Convert from radians to degrees and from degrees to radians.
- ✓ 3126.4.6 Determine the difference made by choice of units for angle measurement when graphing a trigonometric function.
- ✓ 3126.4.7 Find values of inverse trigonometric functions, applying appropriate domain and range restrictions.
- ✓ 3126.4.8 Know and use the following trigonometric identities in verifying other identities: Pythagorean, Reciprocal, Quotient, Sum/Difference, Double Angle
- ✓ 3126.4.9 Know and use the following trigonometric identities in solving trigonometric equations: Pythagorean, Reciprocal, Quotient, Sum/Difference, Double Angle
- ✓ 3126.4.10 Apply the Pythagorean and Reciprocal Identities to verify identities and solve equations.
- ✓ 3126.4.11 Graph functions in polar coordinates.
- ✓ 3126.4.12 Convert between rectangular and polar coordinates.
- ✓ 3126.4.13 Graph the inverse trigonometric functions, identify their key characteristics.
- ✓ 3126.4.14 Graph the six trigonometric function and identify characteristics such as period, amplitude, phase shift, and asymptotes.
- ✓ 3126.4.15 Determine the appropriate domains for each of the inverse trigonometric functions.
- ✓ 3126.4.16 Understand that vectors are determined by the coordinates of their initial and terminal points, or by their components.
- ✓ 3126.4.17 Use vectors to model velocity and direction to solve problems.
- ✓ 3126.4.18 Approximate the area under a curve geometrically by constructing a finite number of rectangles and calculating the total area in those rectangles.
- ✓ 3126.4.19 Compare two different approximations of area under a curve by using a different number of rectangles.

## **Standard 5 – Data Analysis, Statistics, & Probability**

### **Course Level Expectations**

- CLE 3126.5.1 Create scatter plots, analyze patterns and describe relationships that exist in a set of linear and non-linear paired data to model real-world phenomena and make predictions.
- CLE 3126.5.2 Model a data using a variety of transcendental and polynomial models; when possible, determine the best model.
- CLE 3125.5.3 Recognize and explain the potential errors caused by extrapolating from data.

### **Check for Understanding (Formative/Summative Assessment)**

- ✓ 3126.5.1 Explain how to determine the best regression equation model that approximates a particular data set.
- ✓ 3126.5.2 Find the quadratic or exponential regression equations for a data set using a graphing calculator, spreadsheet, and/or estimation.
- ✓ 3126.5.3 Find the equation of the regression line that best fits data with a linear trend.
- ✓ 3126.5.4 Find the regression equation that best fits exponential data.
- ✓ 3126.5.5 Use interpolation to calculate a new data point between two existing data points and identify potential errors.
- ✓ 3126.5.6 Use extrapolation to construct new data points that fit a given trend and identify potential errors.