Integrated Mathematics III 3134

Course description:

This course is the second of three courses in a series that uses a more integrated approach to cover the same algebra and geometry concepts and skills that are included in the traditional three course series. The problem situations, models, and technology used will foster connections among the various strands of mathematics and develop concepts from multiple perspectives.

Standard 1.0: Number and Operations

Students will recognize, represent, model, and apply real numbers and operations verbally, physically, symbolically, and graphically.

Learning Expectations:

The student will:

1.1 demonstrate an understanding of the laws of exponents, including integral and rational exponents;

1.2 demonstrate an understanding of the elements, subsets, and properties of the complex number system.

1.3 select and apply an appropriate method (i.e. mental arithmetic, paper and pencil, or technology) for computing with real numbers, and evaluate the reasonableness of results;

1.4 perform operations on algebraic expression and justify the procedures chosen;

1.5 perform operations on complex numbers of the form a + bi.

Student Performance Indicators:

At Level 1, the student is able to

order a given set of real numbers;

identify the reciprocal of a real number; probe the relationships among various subsets of the real-number system;

explore various representations of absolute value on a number line;

multiply two polynomials with each factor having no more than two terms.

At Level 2, the student is able to

perform basic operations using complex numbers (i.e., addition, subtraction, and multiplication);

identify the exponential form of a logarithmic expression and vice versa;

simplify expressions with rational and negative exponents;

add, subtract, and multiply algebraic expressions.

compare and contrast the GCF and the LCM of a set of algebraic expressions;

add, subtract, and perform scalar multiplication on matrices using appropriate technology.

use the inverse notation of powers and roots;

perform basic operations on rational algebraic expressions.

At Level 3, the student is able to

determine the conjugate of a complex number;

use delta notation to represent the rate of change in a real-world situation

justify the procedures chosen when performing operations on algebraic expressions and equations;

use factorial notation for coefficients in a binomial expansion;
determine the multiplicative inverse of a complex number;
formulate the representation of a series using sigma notation.

Sample Task:

Students design and build a simple fractal from available materials.

Linkages:

Mathematics – Estimation, Measurement, and Computation. Make connections to concept mapping in literature, language arts, and social studies. Connect estimation and computation strategies to business and finance.

Standard 2.0: Algebra

Students will describe, extend, analyze, and create a wide variety of patterns and functions using appropriate materials and representations in real-world problem solving, and will demonstrate an understanding of the behavior of a variety of functions and their graphs.

Learning Expectations:

The student will:

2.1 perform operations on functions, including composition, and determine the effects of the composition on the domain and range;

2.2 demonstrate an understanding of the inverse of a function and determining if the inverse is a function;

2.3 identify and describe the characteristics of families of functions;

2.4 articulate the results of varying parameters of a parent function;

2.5 solve polynomial equations and inequalities using appropriate technology;

2.6 solve absolute value equations and inequalities;

2.7 graph polynomial, exponential, and logarithmic and rational functions;

2.8 solve exponential, logarithmic, and rational equations using appropriate methods and technology;

2.9 solve real-world problems modeled by polynomial, exponential, logarithmic, and periodic functions;

2.10 solve problems involving linear programming;

2.11 demonstrate an understanding of recursive and explicit definitions of functions and sequences;

2.12 recognize the difference between continuous and discrete situations;

2.13 apply sigma notation with arithmetic and geometric series;

2.14 represent a sequence using a list, graph, symbols, and words;

2.15 determine an equation of a conic section from its graph.

Student Performance Indicators:

At Level 1, the student is able to

translate a verbal sentence into an algebraic equation and vice versa;

select the algebraic equation that generalizes the pattern represented by data in a given table;

solve multi-step (more than two steps) linear equations (one set of parentheses on each side of the equations and/or variables on both sides);

select the graph that represents a given linear function expressed in slope-intercept form;

select the graph that models a given real-world situation (i.e., linear and non-linear);

explain what the changes in slope of a non-linear graph represent in a real-world situation;

analyze mathematical patterns related to algebra and geometry in real-world problem solving.

identify the graphical representation of the solution to a one-variable inequality on a number line.

At Level 2, the student is able to

select functional notation to generalize a given numeric pattern;

solve one-variable linear equations with rational expressions;

select the graph of a two-variable inequality;

determine the domain of polynomial, rational, square root, exponential and logarithmic functions;

determine the range of a wide variety of functions given a graph;

solve a system of linear equations with 2 variables (e.g. substitution, elimination, Cramer's Rule, and graphing);

apply properties of logarithms to simplify a logarithmic expression;

identify matrices that model given real-world situations.

use a variety of methods to solve linear systems in two and three variables (e.g., elimination, substitution, Cramer's Rule, matrices, and graphing);

explain the restrictions on the variable in a radical equation;

choose an appropriate method to find the roots of a quadratic equation (e.g. completing the square, quadratic formula, factoring, or graphing calculator);

solve quadratic inequalities.

At Level 3, the student is able to

determine the inverse of a logarithmic function given its graph.

evaluate the graph of a function to determine if it is periodic;

sketch a system of linear inequalities and determine the maximum or minimum value of the related function;

justify the procedures chosen when performing operations on algebraic expressions and equations;

find the maximum or minimum value given the graph of the feasible region of the real world linear programming application;

determine all the roots of a higher order polynomial (i.e., Descartes' Rule of Signs, Rational Root Theorem, and Synthetic Division).

Sample Task:

Examine patterns found in Pascal's Triangle.

Linkages:

Mathematics: Statistics and Probability. Data analysis and pattern recognition in science.

Standard 3.0: Geometry

The student will:

Learning Expectations:

3.1 apply and justify properties of quadrilaterals and circles;

3.2 solve real world problems involving volume of geometric solids;

3.3 demonstrate an understanding of the Platonic Solids;
3.4 demonstrate an understanding of uniqueness through indirect proofs;
3.5 apply transformational matrices to transform geometric figures in a rectangular coordinate system.

Student Performance Indicators:

At Level 1, the student is able to

apply the given Pythagorean Theorem to real-world problems.

At Level 2, the student is able to

predict the graphical transformation that occurs when coefficients and/or constants of given function are changed (no trigonometric or logarithmic functions);

apply proportion and the concepts of similar triangles to solve real world problems;

estimate the irrational solution of a real-world problem using the Pythagorean Theorem.

At Level 3, the student is able to

describe the transformation that has changed a "parent function" to the given related function (e.g., right shift of 3 units, reflection in the x-axis ;

apply the distance formula to obtain the equation of a circle in order to solve real-world problems;

use deductive reasoning to draw conclusions.

use matrices to find the area of a triangle on a coordinate plane;

investigate and explore the conics section.

Sample Task: Students use properties of similar triangles to determine the height of objects that are difficult to measure.

Linkages: Research and discuss geometric applications such as art and use logical reasoning to solve problems in the real world. Use manipulatives to explore the geometric mean of similar triangles; use appropriate tools or technology to develop geometric and spatial concepts; construct three-dimensional objects using physical materials and manipulatives; and compare and construct quadrilateral properties using a variety of models (e.g., Venn diagrams, family trees, manipulative mobiles).

Standard 4.0: Measurement

The student will:

4.1 use concepts of length, area, and volume to estimate and solve real-world problems;

4.2 apply measurement concepts and relationships in algebraic and geometric problem-solving situations;

4.3 use estimation to make predictions and determine reasonableness of results;

4.4 demonstrate an understanding of rates and other derived and indirect measurements (e.g. velocity, miles per hr, rpm, cost per unit).

Student Performance Indicators:

At Level 1, the student is able to

- select the appropriate unit of measure given the real world situation.

select the area representation for a given product of two binomials.

At Level 2, the student is able to

apply the given formula to find area and circumference of circles, area and perimeter of polygons, and volume of regular solids;

use appropriate measurements in collecting data for a real world situation

At Level 3, the student is able to

solve real world problems given logarithmic and exponential formulas (e.g. Ph scale, Richter scale.).

Sample Task:

Construct a regular geometric solid and determine the surface area, volume, and edge length.

Linkages:

Science, art, construction, manufacturing.

Standard 5.0: **Data Analysis and Probability**

Students will collect, organize, represent, and interpret data; make and evaluate inferences and predictions; present and evaluate arguments based on data analysis; and model situations to determine theoretical and experimental probabilities.

Learning Expectations:

The student will:

5.1 describe and apply the normal distribution and its properties;

5.2 use z-scores to compare normally distributed data sets;

5.3 use a variety of techniques to determine equations of best fit for nonlinear data sets;

5.4 calculate and interpret z-scores;

5.5 apply the properties of conditional probability;

5.6 determine binomial probabilities using appropriate methods;

5.7 make inferences about a data set using appropriate measures of central tendency and dispersion, including variance and standard deviation;

5.8 calculate expected value to make judgments about real-life situations.

Student Performance Indicators:

At Level 1, the student is able to

make a prediction from the graph of a real-world data set;

determine the measures of central tendency for a given set of real-world data;

choose the matching linear graph when given a set of ordered pairs representing real-world data.

analyze student-collected data to make predications or generalizations.

At Level 2, the student is able to

categorize the correlation of a scatterplot using real-world data (i.e., positive, negative, strong, or weak);

determine the number of possible outcomes for a given experiment (i.e. the multiplication counting principle, permutations, or combinations);

determine the theoretical probability of a simple event for a given situation;

use simulations to help predict the probability of a given situations;

determine the theoretical probability of a compound event (i.e., dependent or independent, union and intersection);

determine the theoretical probability of mutually exclusive events for a given situation;

analyze theoretical or experimental probability to determine the likelihood of an event;

analyze data using linear and quadratic functions using the appropriate technology;

analyze the validity of statistical conclusions and the use, misuse, and abuse of data;

identify the mean and the standard deviation given the graph of a normal distribution.

At Level 3, the student is able to

find the equation for the line of best fit given a scatterplot depicting real-world data

use the measure of central tendency which best represents the given real-world data set given a distribution curve.

Sample Task:

Students analyze real-world data collected from the newspaper and explore and report the uses, misuses, and abuses of reported statistical data. Students search the internet to collect age and market value of a selected vehicle over a specific period of time. They use a graphing calculator to create a scatterplot and construct a line of best fit to predict the depreciation of the vehicle.

Linkages:

Business and Economics; Sports; Social Studies, Science.