TCAP Mathematics Calculator Prohibited Subpart (Subpart I)

The TCAP mathematics calculator prohibited subpart (currently subpart 1) is designed to measure number sense, conceptual understanding, and fluency. The overarching goal is to measure how efficiently students work with mathematics without a calculator. In order to be successful, students need to be equipped with a deep, conceptual understanding of Tennessee's grade-level standards, strong number sense, and strategies that allow them to work mathematics fluently. This document includes **frequently asked questions** and **key concepts and standards connections** to clarify important information about the mathematics calculator prohibited subpart.

Frequently Asked Questions

1. ***How are the expectations in the calculator prohibited subpart aligned to the Tennessee Academic Standards?***
   The calculator prohibited subpart of all mathematics TCAP assessments focuses on assessing procedural and computational fluency, number sense, and conceptual understanding, which are addressed in the Tennessee academic standards. See **Key Concepts and Standards Connections.**

2. ***Why is there a time limit for the calculator prohibited subpart?***
   A time limit is necessary to fully assess procedural and computational fluency, number sense, and conceptual understanding. Students must work within a time limit to demonstrate how they can rely on these skills to efficiently work with mathematics without a calculator.

3. ***Will all students have enough time to complete the calculator prohibited subpart of TCAP?***
   The test includes items that represent a broad range of difficulty to provide all students, including the highest achieving students, the opportunity to show growth. This means that all students may not finish. Keep in mind, though, that students may find that they did well on the calculator prohibited subpart, even if they did not answer every question.
   It is also important to note that some items on the assessment can be solved using strategies other than pure calculation. On all calculator prohibited subparts there are items that require students to employ number sense and demonstrate an understanding of standards that require students to think about the relationships that exist between numbers without actually performing calculations. Additionally, there are standards, particularly in the measurement and geometry domains, that do not inherently have students' access computation skills. For grades 3-7, 30-50 percent of the items in the calculator prohibited subpart require this type of thinking or do not have a computational focus. In grade 8 through the EOC courses, 40-60 percent of the items on the calculator prohibited subpart are designed this way. Additionally, within the calculator prohibited subpart, students are typically asked to work with friendly numbers. With a strong grasp of conceptual understanding, students can successfully work these problems without completing calculations. Please see the example* on the following page:

Which expression has a value greater than 32,978?

A. $32{,}978 \times \frac{1}{2}$

B. $32{,}978 \times \frac{7}{3}$

C. $32{,}978 \times \frac{5}{16}$

D. $32{,}978 \times \frac{8}{8}$

Please note: this is not an actual test item; this example was created for illustrative purposes.

4. ***How is proficiency determined?***
   Tennessee educators completed standard setting this summer. During standard setting, educators reviewed actual test items and recommended the cut scores for each performance level descriptor (below, approaching, on track, mastered) for the assessments. The standard setting committees will share their recommendations with the State Board of Education for approval.

   We will continue to listen to educator feedback and monitor student performance and assessment times to ensure we are striking the right balance. When we move to online assessments, we will have even better data about the time students spend on each test item and each subpart of the assessment. We are committed to continuously improving how we communicate expectations around the calculator prohibited portion of the mathematics assessments, as well as providing teachers with the resources they need to ensure students are prepared to show what they know and are able to do.

Key Concepts and Standards Connections

TCAP mathematics subpart 1 focuses on assessing:

- Conceptual understanding
- Fluency (both procedural and computational)
- Number sense

The standards set expectations related to these concepts, both in the narrative of the standards document and embedded in the standards themselves. The calculator prohibited subpart is focused on assessing that students have a comprehensive understanding of their grade level standards and thorough ability to demonstrate fluency, number sense, and conceptual understanding as defined below. With these foundations in place, students will be prepared to be successful on the calculator prohibited subpart of TCAP.

Conceptual Understanding

The introduction for the Tennessee Academic Standards for mathematics describes conceptual understanding as the following:

> *"Conceptual understanding refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts."*

The following chart provides standards and examples of items that are assessing a student's conceptual understanding of the Tennessee mathematics standards.

Conceptual Understanding
Example 1: Which number goes in the box to make the equation true? $\frac{\square}{3} = 1$ A. 1 B. 3 C. 6 D. 9 Standard Alignment: 3.NF.A.3c
Example 2: Triangle A'B'C' is created by using only rigid motion transformations of triangle ABC. Which transformations could have been used to create triangle A'B'C'? A. A dilation of 4 centered at the origin followed by a translation down 2 units B. A reflection across the x-axis followed by a dilation of 3 centered at the origin C. A rotation of 180 degrees counterclockwise about the origin followed by a dilation of 2 centered at the origin D. A rotation of 90 degrees clockwise followed by a translation of 2 units to the right Standard Alignment: G.CO.B.6 and M1.G.CO.B.6
Example 3: Which statement describes a unit rate? A. Amy ran 5 miles B. Amy ran 5 miles in 1 hour C. Amy ran 10 miles in 2 hours D. Amy ran 10 miles and then rested for 1 hour Standard Alignment: 6.RP.A.2

Example 4:

An equation is shown.
$a - b = a - c$

Which statement is true?

A. $b < c$
B. $b > c$
C. $b = -c$
D. $-b = -c$

Standard Alignment: 7.NS.A.3

Fluency

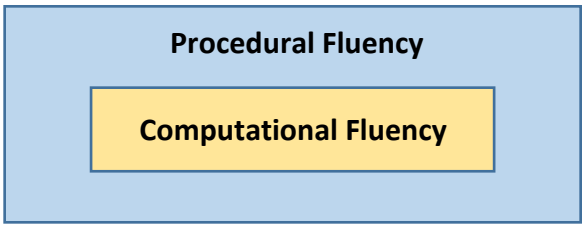
The Tennessee Academic Standards introduction emphasizes fluency as a key characteristic of mathematically prepared students:

> *"All students should be able to recall and use their math education when the need arises. That is, a student should know certain math facts and concepts such as the multiplication table, how to add, subtract, multiply, and divide basic numbers, how to work with simple fractions and percentages, etc. There is a level of procedural fluency that a student's K-12 math education should provide him or her along with conceptual understanding so that this can be recalled and used throughout his or her life. Students also need to be able to reason mathematically. This includes problem solving skills in work and non-work related settings and the ability to critically evaluate the reasoning of others."*

The mathematics standards introduction further elaborates fluency expectations:

> *"Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly. One cannot stop with memorization of facts and procedures alone. It is about recognizing when one strategy or procedure is more appropriate to apply than another. Students need opportunities to justify both informal strategies and commonly used procedures through distributed practice. Procedural fluency includes computational fluency with the four arithmetic operations. In the early grades, students are expected to develop fluency with whole numbers in addition, subtraction, multiplication, and division. Therefore, computational fluency expectations are addressed throughout the standards. Procedural fluency extends students' computational fluency and applies in all strands of mathematics. It builds from initial exploration and discussion of number concepts to using informal strategies and the properties of operations to develop general methods for solving problems (NCTM, 2014)."*

Procedurally fluency is not only referencing the ten standards explicitly addressing computational fluency; holistically, fluency is the ability to calculate efficiently in all situations. **Both** computational and procedural fluency are assessed on the calculator prohibited subpart of all TCAP mathematics assessments.



Examples of Standards with Embedded Procedural Fluency*	
3.OA.A.3	Multiply and divide within 100 to solve contextual problems, with unknowns in all positions, in situations involving equal groups, arrays, and measurement quantities using strategies based on place value, the properties of operations, and the relationship between multiplication and division.
4.NBT.B.5	Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between operations; assess the reasonableness of answers using estimation strategies. (Limit division problems so that either the dividend or the divisor is a whole number.)
6.NS.A.1	Interpret and compute quotients of fractions, and solve contextual problems involving division of fractions by fractions (e.g., using visual fraction models and equations to represent the problem is suggested).
7.EE.A.2	Understand that rewriting an expression in different forms in a contextual problem can provide multiple ways of interpreting the problem and how the quantities in it are related.
8.EE.C.8b	Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
A1.A.REI.B.3b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions.
M2.F.IF.B.5a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
M3.G.GPE.B.3	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

**This is a small group of procedural fluency standards. There are many more standards in each grade/course that have inherent aspects of procedural fluency. The following page has additional standards with examples provided.*

Procedural Fluency

Example 1:

An expression is shown.

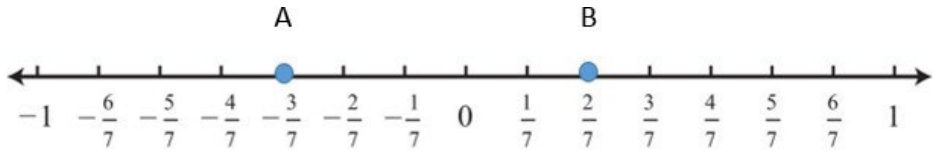
$$\frac{1}{3}+\frac{5}{6}+\frac{2}{12}$$

What is the value of the expression?

Standard Alignment: 5.NF.A.1

Example 2:

A number line is shown.



What is the distance between points A and B?

Standard Alignment: 7.NS.A.1

Example 3:

A system of equations is shown.

$y = 3x - 2$
$y = 3x - 4$

How many solutions does this system of equations have?

A. 0
B. 1
C. 2
D. An infinite number

Standard Alignment: 8.EE.C.7a

Example 4:

A diver jumps from a platform above the water. When the diver leaves the platform, he has an initial upward velocity. The equation represents the height, *h(t)*, in meters of the diver's feet above the water *t* seconds after he jumps from the platform.

$$h(t) = -4.9t^2 + 6t + 10$$

What is the height, in meters (m), of the springboard above the water?

Standard Alignment: A1.F.IF.B.3 and M2.F.IF.A.1

Computational Fluency Standards	
K.OA.A.5	Fluently add and subtract within 10 using mental strategies.
1.OA.C.6	Fluently add and subtract within 20 using mental strategies. By the end of 1st grade, know from memory all sums up to 10.
2.OA.B.2	Fluently add and subtract within 30 using mental strategies. By the end of 2nd grade, know from memory all sums of two one-digit numbers and related subtraction facts.
2.NBT.B.5	Fluently add and subtract within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction.
3.OA.C.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of 3rd grade, know from memory all products of two one-digit numbers and related division facts.
3.NBT.A.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
4.NBT.B.4	Fluently add and subtract within 1,000,000 using appropriate strategies and algorithms.
5.NBT.B.5	Fluently multiply multi-digit whole numbers (up to three-digit by four-digit factors) using appropriate strategies and algorithms.
6.NS.B.2	Fluently divide multi-digit numbers using a standard algorithm.
6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.

The following page has additional standards with examples provided.

Computational Fluency

Example 1:

An expression is shown.
$8 \div 4$

What is the value of the expression?

Standard Alignment: 3.OA.C.7

Example 2:

An expression is shown.

3500 x 21

What is the value of the expression?

Standard Alignment: 5.NBT.B.5

Example 3:

What is the sum?
$23{,}405 + 9{,}995$

Standard Alignment: 4.NBT.B.4

Example 4:

What is the product of -2 and -12?

Note: While the 7th grade standard addressed in this item is not listed as a computational fluency standard, computing with rational numbers is important and embedded within the 7th grade standards.

Standard Alignment: 7.NS.A.2c

Number Sense

Number sense is a student's ability to work fluidly and flexibly with numbers. A student with strong number sense can:

- Demonstrate a strong sense of the meaning of numbers
- Identify relationships between numbers
- Assess the reasonableness of answers
- Perform mental calculations
- Understand how numbers are taken apart and put together in multiple ways
- Make connections between the operations
- Use numbers in real world situations

Examples of Number Sense Standards**	
3.NF.A.3d	Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Use the symbols >, =, or < to show the relationship and justify the conclusions.
4.NBT.A.1	Recognize that in a multi-digit whole number (less than or equal to 1,000,000), a digit in one place represents 10 times as much as it represents in the place to its right. *For example, recognize that 7 in 700 is 10 times bigger than the 7 in 70 because 700 ÷ 70 = 10 and 70 x 10 = 700.*
5.NF.B.5b	Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product less than the given number; and relate the principle of fraction equivalence $\frac{a}{b} = \frac{a\ x\ n}{b\ x\ n}$ to the effect of multiplying $\frac{a}{b}$ by 1.
6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real world contexts, explaining the meaning of 0 in each situation.
7.NS.A.1c	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers locating them approximately on a number line diagram. Estimate the value of irrational expressions such as π^2.
A1.N.Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
A2.S.ID.A.1	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages using the Empirical Rule.

***This is a small group of the number sense standards. There are other standards in all grades/courses that inherently employ number sense. The following page has additional standards with examples provided.*

Number Sense

Example 1:

Decide if each number sentence is true or false. Mark the correct box in the table for each number sentence.

	True	False
$\frac{2}{3} < \frac{5}{6}$	O	O
$\frac{3}{8} < \frac{1}{2}$	O	O
$\frac{2}{4} < \frac{5}{10}$	O	O

Standard Alignment: 4.NF.A.2

Example 2:

An expression is shown.

30 + 40

Julio uses the greatest common factor to create an equivalent expression.

Which is Julio's expression?

A. 2(15 + 20)
B. 10(3 + 4)
C. 5(6 + 8)
D. 10(3 + 40)

Standard Alignment: 6.NS.B.4

Example 3:

Select all the numbers that are irrational.

A. 0.50
B. π
C. $\frac{1}{3}$
D. $\sqrt{9}$
E. $\sqrt{11}$

Standard Alignment: 8.NS.A.1

Example 4:

Which number when raised to the power of 3 equals 2?

A. $\frac{1}{3}$
B. $\frac{2}{3}$
C. $2^{\frac{1}{3}}$
D. $8^{\frac{1}{3}}$

Standard Alignment: A2.N.RN.A.1 or M2.N.RN.A.1