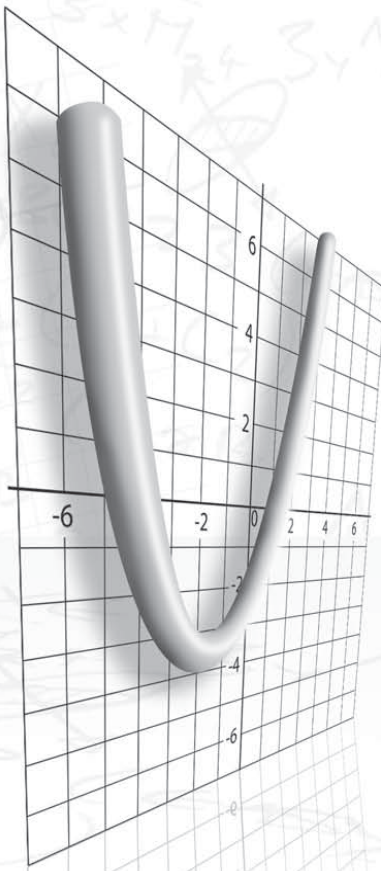
Student Name _____

Teacher Name _____

School _____

System _____



# ALGEBRA II

# Item Sampler

## Tennessee End of Course Assessment
## Algebra II Form 4

## Reporting Category 4:
## Geometry and Measurement

PEARSON

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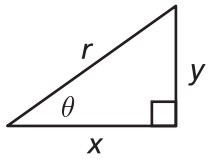
# Algebra II Reference Page

## Trigonometric Functions

$\sin\theta = \frac{y}{r}$, $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$, $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$, $\cot\theta = \frac{x}{y}$




$r = \sqrt{x^2 + y^2}$

## Logarithm Properties

$\log_b MN = \log_b M + \log_b N$

$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

$\log_b M^p = p \log_b M$

$\log_b x = y \Leftrightarrow x = b^y$

## Arithmetic and Geometric Sequences and Series

$a_1 = 1^{\text{st}}$ term $\quad r =$ common ratio $\quad d =$ common difference

$a_n = n^{\text{th}}$ term $\quad n =$ number of terms in series

Arithmetic Sequence: $a_n = a_1 + (n-1)d$ Geometric Sequence: $a_n = a_1 r^{n-1}$

Sum of a Finite Arithmetic Series: $S_n = \frac{n(a_1 + a_n)}{2}$ or $S_n = \frac{1}{2}n[2a_1 + (n-1)d]$

Sum of a Finite Geometric Series: $S_n = \frac{a_1(1-r^n)}{1-r}$, $r \neq 1$

Sum of an Infinite Geometric Series: $S = \frac{a_1}{1-r}$ where $|r| < 1$

## Combinations

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

## Permutations

$${}_nP_r = \frac{n!}{(n-r)!}$$

## Binomial Theorem

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$

## Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = ax^2 + bx + c$$

## Interest Formulas

Compound interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

Continuous compound interest: $A = Pe^{rt}$

$P =$ present value

$A =$ future value

$r =$ annual interest rate

$t =$ time in years

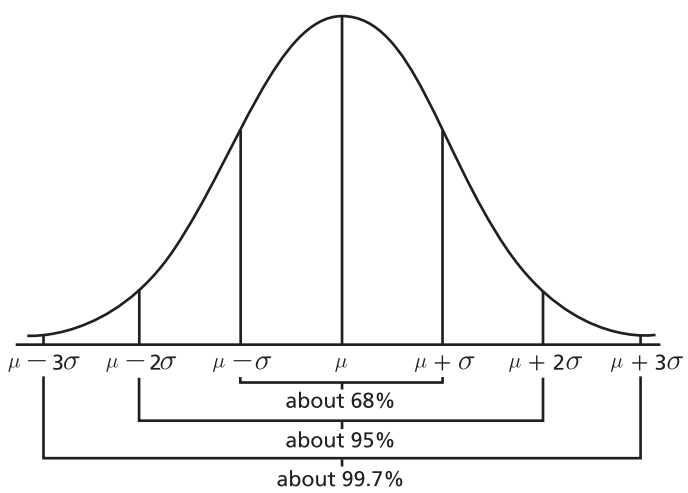
$n =$ frequency of compounding per year

# Algebra II Reference Page

## Conic Sections – Standard Equations

| | | | |
|---|---|---|---|
| Parabola | $y = a(x-h)^2 + k$ | or | $x = a(y-k)^2 + h$ |
| | $(y-k)^2 = 4p(x-h)$ | or | $(x-h)^2 = 4p(y-k)$ |
| Circle | $(x-h)^2 + (y-k)^2 = r^2$ | | |
| Ellipse | $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ | or | $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ |
| Hyperbola | $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ | or | $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ |

## Normal Curve Distribution




## Standard Deviation

The standard deviation , $\sigma$, for values $x_1, x_2, x_3, \ldots, x_n$ with mean $\mu$ is determined by the following:

$$\sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \ldots + (x_n - \mu)^2}{n}}$$

## Probability Formulas

Exclusive

$$P(A \text{ or } B) = P(A) + P(B)$$

Inclusive

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Independent

$$P(A \text{ and } B) = P(A) \bullet P(B)$$

Dependent

$$P(A \text{ and } B) = P(A) \bullet P(B|A)$$

Conditional

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

# Algebra II Reference Page

## Cramer's Rule for Solving a System of Linear Equations

For a $2 \times 2$ System:

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

For a $3 \times 3$ System:

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

## Converting Degrees to Radians

Multiply degree measure by $\frac{\pi}{180°}$

## Converting Radians to Degrees

Multiply radian measure by $\frac{180°}{\pi}$

## Definition of "$i$"

$$i^2 = -1$$
$$i = \sqrt{-1}$$

## Absolute Value of a Complex Number

$$|a + bi| = \sqrt{a^2 + b^2}$$

# Contents

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# Introduction to Algebra II

## Content of tests

The testing program titled the *Tennessee End of Course Assessment* was established to meet the Tennessee mandate for end of course assessments in Tennessee secondary schools. These tests measure the Tennessee State Performance Indicators. Subject areas covered by the end of course assessments include Mathematics, Language Arts, History, and Science.

## Test development

For the *Tennessee End of Course Assessment*, a staff of writers – composed of both teachers and professional test developers experienced in each of the content areas – researched and wrote the items. Professional editors and content specialists carefully reviewed all items and test directions for content and accuracy. To provide a large pool of items for final test selection, the test developers created approximately twice as many items as were needed in the final editions of the tests.

After tryout tests were administered, student responses were analyzed. Professional content editors and researchers carefully reviewed items, their data, and test directions for content, suitability, and accuracy before including particular items and test directions in operational tests.

## Test administration

*Tennessee End of Course Assessments* are given to students as they near the end of courses that are included in the program. Tests may be given midyear for block schedules or at the end of the school year.

This test contains 65 multiple-choice questions.

You will have ample time to read and answer each of the questions. The Algebra II test has been designed to be administered in one session and is not timed. The first 15 minutes are set aside to complete identifying data on the answer sheet.

Calculator use is optional. Sharing calculators during testing is not permitted.

The following types of calculators/devices may **NOT** be used during the test:

- pocket organizers
- electronic writing pads or input devices
- Some examples of prohibited calculators are:
    - Casio models: CFX-9970G, Algebra FX 2.0
    - Hewlett-Packard models: HP-40G, HP-49G
    - Texas Instruments models: TI-89, TI-92, Voyage 200, TI-NSPIRE – the CAS version (The non-CAS version of TI-NSPIRE is allowable.)
- calculators that can communicate (transfer data or information) wirelessly with other student calculators/devices
- cell phones, PSPs, and/or iPods
- Students may use any four-function, scientific, or graphing calculator does not have any of the above features. The use of units that have a Computer Algebra System (CAS) is NOT allowed.

# Tips for Taking the Test

**Preparing for the test**

- Review this Tennessee End of Course Item Sampler for Algebra I carefully and thoroughly.
- Acquire the Tennessee End of Course Practice Test for Algebra I, and take the test several times.
- Become familiar with the correct way to mark answers on the answer sheet. There is a sample answer sheet in this Practice Test.

**Before the test**

- Get a good night's sleep. To do your best, you need to be rested.

**During the test**

- Relax. It is normal to be somewhat nervous before the test. Try to relax and not worry.
- Listen. Listen to and read the test directions carefully. Ask for an explanation of the directions if you do not understand them.
- Plan your time. Do not spend too much time on any one question. If a question seems to take too long, skip it and return to it later. First answer all questions that you are sure about.
- Think. If you are not sure how to answer a question, read it again and try your best to answer the question. Rule out answer choices that you know are incorrect and choose from those that remain.

# Directions for Using the Item Sampler

This Item Sampler for Algebra II provides specific information to students and teachers. It contains examples of different item types for each Performance Indicator that may be tested in any given end of course test administration. Performance Indicators have been grouped by Reporting Categories. These Reporting Categories will be used to report information regarding performance on the end of courts test to students, teachers, schools, and systems.

The items in this Item Sampler will not be found in the end of course tests. The number of items in this Item Sampler does not reflect the emphasis of content on the test. In order to identify the emphasis of content, the End of Course Assessment Practice Test for Algebra I should be used. The Practice Test gives a better representation of content emphasis across Reporting Categories and Performance Indicators.

An Answer Key is located in Page 20. Use it to check your answers. Review items that you get wrong.

## Reporting Category: Geometry and Measurement

**Numbers 1 through 15**

**Performance Indicator:** 3103.4.1 Exhibit knowledge of unit circle trigonometry.

1. **What is the value of $\csc\left(\frac{7\pi}{6}\right)$?**

- **A** $-2$
- **B** $2$
- **C** $-\frac{2\sqrt{3}}{3}$
- **D** $\frac{2\sqrt{3}}{3}$

**Performance Indicator:** 3103.4.1 Exhibit knowledge of unit circle trigonometry.

2. **Which expression is equivalent to sin(210°)?**

- **A** sin(−60°)
- **B** sin(60°)
- **C** sin(−30°)
- **D** sin(30°)

Go On ▶

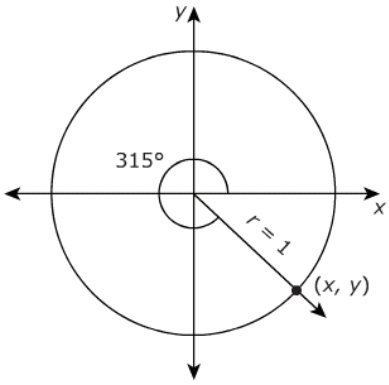
**Performance Indicator:** 3103.4.1 Exhibit knowledge of unit circle trigonometry.

3. **What is the degree equivalent to $\frac{9\pi}{15}$ radians?**

- **A** 54°
- **B** 108°
- **C** 216°
- **D** 300°

**Performance Indicator:** 3103.4.1 Exhibit knowledge of unit circle trigonometry.

4. **The angle shown below is in standard position.**




**What is the value of *x*?**

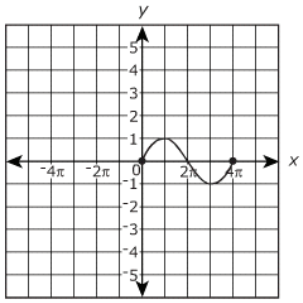
- **A** $-\sqrt{2}$
- **B** $-\frac{\sqrt{2}}{2}$
- **C** $\frac{\sqrt{2}}{2}$
- **D** $\sqrt{2}$

Go On ▶

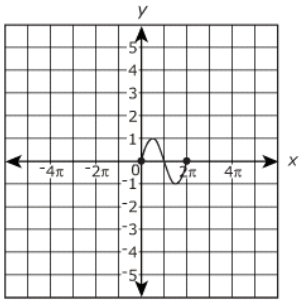
**Performance Indicator:** 3103.4.2 Match graphs of basic trigonometric functions with their equations.

5. **Which graph <u>best</u> represents one cycle of $y = \sin\left(\frac{x}{2}\right)$?**

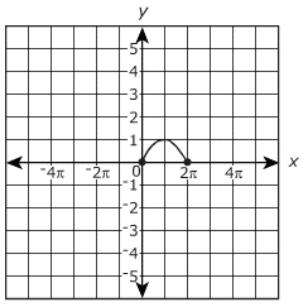
○ **A**



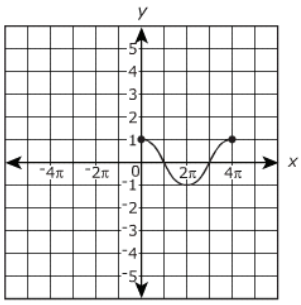

○ **C**




○ **B**

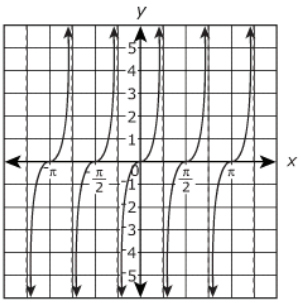



○ **D**




**Performance Indicator:** 3103.4.2 Match graphs of basic trigonometric functions with their equations.

6. **Which trigonometric function <u>best</u> describes the graph below?**




○ **A** $y = 2\tan x$

○ **B** $y = \tan 2x$

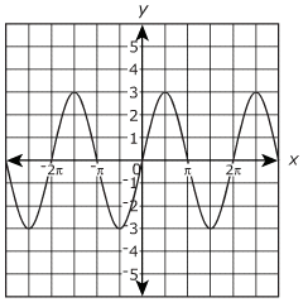
○ **C** $y = \frac{1}{2}\tan x$

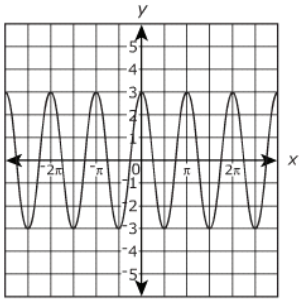
○ **D** $y = \tan\left(\frac{1}{2}x\right)$

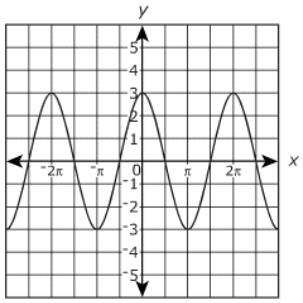
Go On ▶

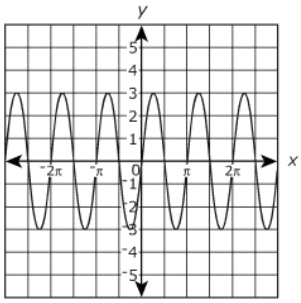
**Performance Indicator:** 3103.4.2 Match graphs of basic trigonometric functions with their equations.

7. **Which graph best represents the function $y = 3\sin x$?**

A


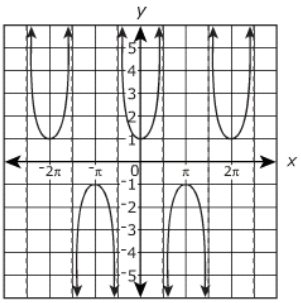

C



B



D



**Performance Indicator:** 3103.4.2 Match graphs of basic trigonometric functions with their equations.

8. **Which trigonometric function is represented by the graph below?**




A $y = \cot x$

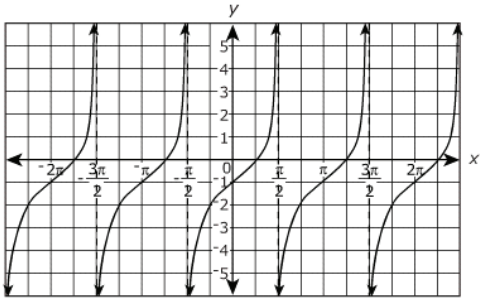
B $y = \tan x$

C $y = \sec x$

D $y = \csc x$

**Performance Indicator:** 3103.4.2 Match graphs of basic trigonometric functions with their equations.

9. **A trigonometric function is graphed below.**



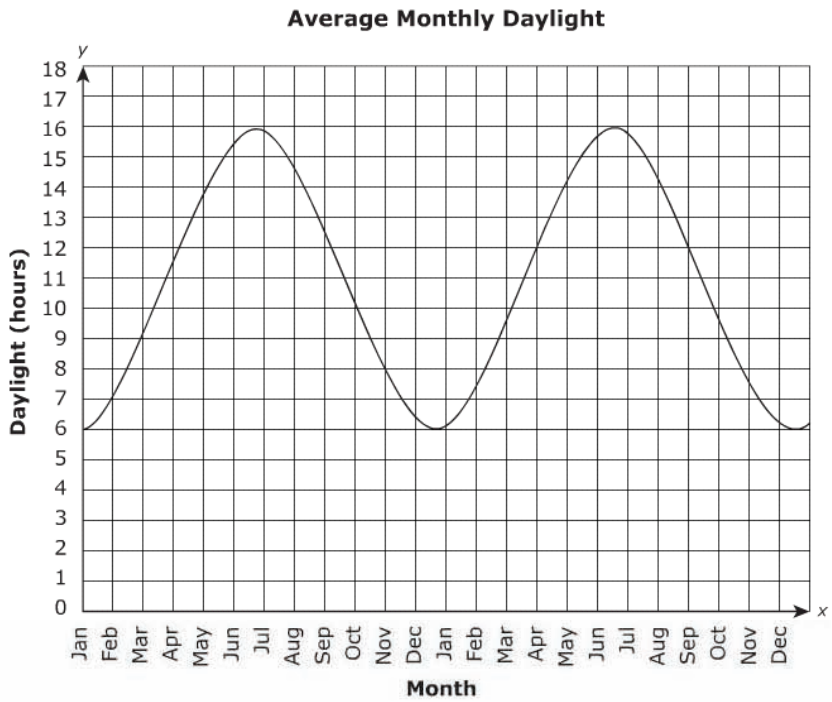

**Which function is <u>best</u> represented by this graph?**

- **A** $f(x) = \cot(x) - 1$
- **B** $f(x) = \tan(x) - 1$
- **C** $f(x) = \cot(x - 1)$
- **D** $f(x) = \tan(x - 1)$

**Performance Indicator:** 3103.4.3 Describe and articulate the characteristics and parameters of parent trigonometric functions to solve contextual problems.

10.

**The following graph shows the average monthly daylight in a particular city over two years. The function describing this graph is a transformation of the parent sine function $y = \sin x$.**

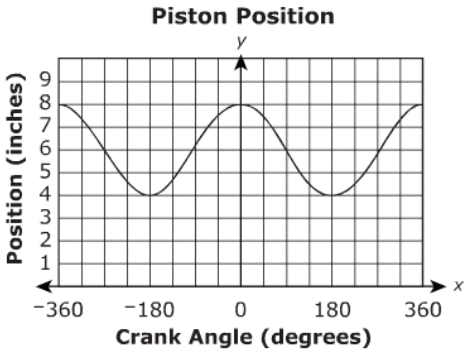



**Which value best describes the amplitude of the transformed function?**

- **A** 5 hours
- **B** 6 hours
- **C** 10 hours
- **D** 16 hours

**Performance Indicator:** 3103.4.3 Describe and articulate the characteristics and parameters of parent trigonometric functions to solve contextual problems.

11. **In a crankshaft system, the position of the piston with respect to the crank angle can be modeled by a cosine function.**

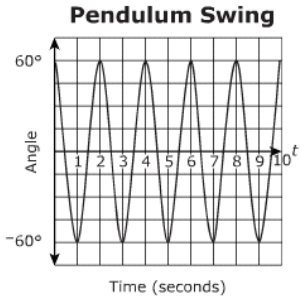



**What is the period and amplitude of the oscillation?**

- **A** period = 180°; amplitude = 4 inches
- **B** period = 360°; amplitude = 4 inches
- **C** period = 180°; amplitude = 2 inches
- **D** period = 360°; amplitude = 2 inches

**Performance Indicator:** 3103.4.3 Describe and articulate the characteristics and parameters of parent trigonometric functions to solve contextual problems.

12. **The bob of a simple pendulum is released from a point, and its swing is modeled by a cosine function.**

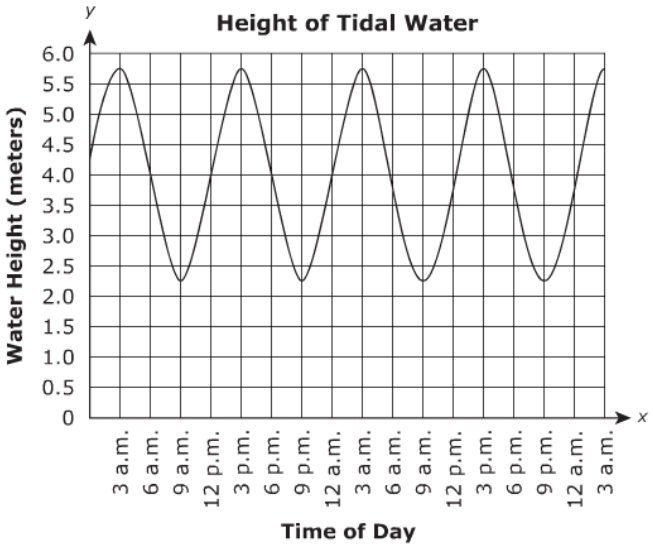



**What is the period, in seconds, of the oscillation?**

- **A** 1
- **B** 2
- **C** 4
- **D** 10

**Performance Indicator:** 3103.4.3 Describe and articulate the characteristics and parameters of parent trigonometric functions to solve contextual problems.

13. **The graph below shows the height of tidal water in the same location for two consecutive days.**

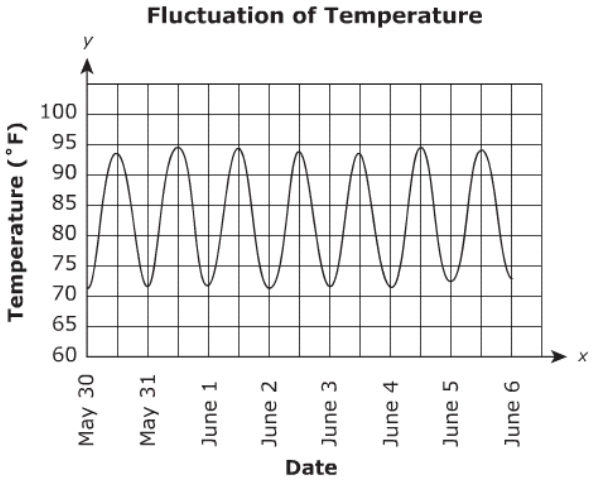



**The function describing this graph is a transformation of the parent sine function $y = \sin x$. Which value is closest to the amplitude of this transformed function?**

- **A** 1.7 meters
- **B** 2.3 meters
- **C** 4.0 meters
- **D** 5.7 meters

**Performance Indicator:** 3103.4.3 Describe and articulate the characteristics and parameters of parent trigonometric functions to solve contextual problems.

14. **The graph below shows the fluctuation of temperature in a city over a one-week period.**



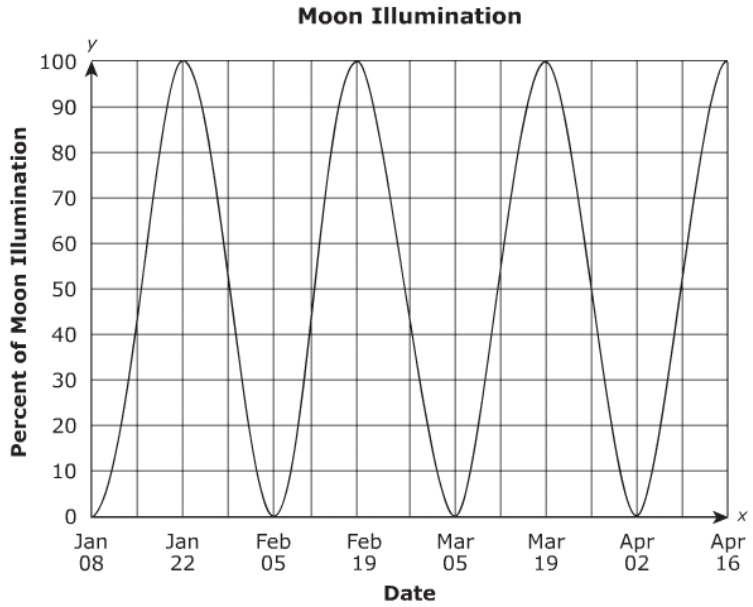

**The function describing this graph can be modeled by the transformation of the parent sine function $y = \sin x$. Which is closest to the amplitude?**

- **A** 11°F
- **B** 22°F
- **C** 72°F
- **D** 83°F

Go On ▶

**Performance Indicator:** 3103.4.3 Describe and articulate the characteristics and parameters of parent trigonometric functions to solve contextual problems.

15. **The percent of illumination of the moon as seen from Earth can be modeled by a sine function.**




**Which is closest to the period of the function?**

- **A** 7 days
- **B** 14 days
- **C** 22 days
- **D** 28 days

| Reporting Category 4: Geometry and Measurement | | |
|---|---|---|
| **Item Number** | **Correct Answer** | **Performance Indicator** |
| 1 | A | 3103.4.1 Exhibit knowledge of unit circle trigonometry. |
| 2 | C | 3103.4.1 Exhibit knowledge of unit circle trigonometry. |
| 3 | B | 3103.4.1 Exhibit knowledge of unit circle trigonometry. |
| 4 | C | 3103.4.1 Exhibit knowledge of unit circle trigonometry. |
| 5 | A | 3103.4.2 Match graphs of basic trigonometric functions with their equations. |
| 6 | B | 3103.4.2 Match graphs of basic trigonometric functions with their equations. |
| 7 | A | 3103.4.2 Match graphs of basic trigonometric functions with their equations. |
| 8 | C | 3103.4.2 Match graphs of basic trigonometric functions with their equations. |
| 9 | B | 3103.4.2 Match graphs of basic trigonometric functions with their equations. |
| 10 | A | 3103.4.3 Describe and articulate the characteristics and parameters of parent trigonometric functions to solve contextual problems. |
| 11 | D | 3103.4.3 Describe and articulate the characteristics and parameters of parent trigonometric functions to solve contextual problems. |
| 12 | B | 3103.4.3 Describe and articulate the characteristics and parameters of parent trigonometric functions to solve contextual problems. |
| 13 | A | 3103.4.3 Describe and articulate the characteristics and parameters of parent trigonometric functions to solve contextual problems. |

| | | |
|---|---|---|
| 14 | A | 3103.4.3 Describe and articulate the characteristics and parameters of parent trigonometric functions to solve contextual problems. |
| 15 | D | 3103.4.3 Describe and articulate the characteristics and parameters of parent trigonometric functions to solve contextual problems. |