PBS Lesson Series

Math: Grade 8, Lesson 13, Analyze and Solve Systems of Linear Equations

Lesson Focus: Solving Systems by Substitution
Practice Focus: Students will focus on understanding that substitution is a useful method for solving a system of linear equations. Students will work on rewriting one of the equations for one variable in terms of the other, substituting that expression into the other equation, and then solving algebraically.
Objective:

- Understand how substitution can be used to solve a linear system of equations
- Apply this understanding to interpret the results with one solution, no solutions, or infinitely many solutions

Key Vocabulary: System of linear equations**,** solution of a system of linear equations, substitution method**,** one solution**,** no solutions**,** infinitely many solutions
TN Standards: 8.EE.C.8
Teacher Materials:

- Whiteboard and Markers
- Student Practice Packet

Student Materials:

- Paper and a pencil, and a surface to write on
- Calculator not required but may be used to check calculations.

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Week 3 of Tennessee's At Home Learning Series for math! Today's lesson is for all our 8th graders out there, though all students are welcome to tune in. This lesson is the thirteenth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. If you don't already have the student packet for this lesson, you can find it online at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.** **Today we will be learning about systems of linear equations, and we will be working on estimating solutions by inspection! Before we get started, to participate fully in our lesson today, you will need:** • **Paper and a pencil, and a surface to write on** • **A calculator is not required but may be used to check calculations.**	Students get materials ready for the lesson.

Ok, let's begin!	
Intro (2 min) **You may recall that in Lesson 11 and 12, we used this chart about maximum heart rates based on a person's age.** [Teacher show the chart.] **Heart Rates (beats per minute)** **Maximum Heart Rate (MHR)** — **220 - age** **Vigorous Intensity Exercise** — **70-85% of MHR** **Moderate Intensity Exercise** — **50-70% of MHR** **Remember that x represents the age of a person, and y represents the maximum heart rate. Here's what we wrote in Lessons 11 and 12:** [Write on whiteboard.] $y = 220 - x$ **In this equation, x represents the age of a person, and *y* represents the maximum heart rate.** **Let's work another problem related to this same chart. Give two ordered pairs that are solutions to the equation** $y = 0.8(220 - x)$ **How can you find an ordered pair that is a solution to the equation? [pause] You can pick a value for *x*, substitute it in the equation, and solve for *y*.** **The heart rate that is 80% of the maximum heart rate of a person who is 20 years old is 160 beats per minute. The heart rate that is 80% of maximum heart rate of a person who is 40 years old is 144 beats per minute. Therefore, the ordered pairs are (20, 160) and (40, 144).** [Teacher write/show and speak.] **How does the information in the table help us to interpret the meaning of the equation and the ordered pairs? The table shows that 80% of maximum heart rate, which is 220 minus the person's age** [Teacher point to information in chart.] **So the ordered pairs give 80% of the MHR, or a vigorous exercise MHR, for two specific ages. This equation represents a function. There are many different ways a function can be represented, such as tables, graphs and descriptions.**	Students listen to the problem, consider what the problem is asking, and determines possible solution strategies.

Let's look at some other problems. Are you ready? [Pause] **OK, let's get started!**	
Teacher Model (10-12 min) Objective 1: Example 1 – Understand when a substitution is a useful method for solving systems of equations for one solution. **Using substitution to solve a system of equations is a useful method when it is easy to isolate one variable in one or both of the equations. For instance, listen to this problem:** [Teacher read/show problem.] **Gemma sells tickets at a local fair. Adult tickets are $13.50 each while children's tickets are $7.50 each. On Saturday, she sells 800 tickets and collects a total of $7,680. How can Gemma determine the number of each type of ticket that was sold on Saturday?** **Now that we've heard the problem, get out your pencil and paper, and let's write a system of linear equations to represent the situation. We know that the number of children's tickets plus the number of adult tickets equals the total tickets. If *c* = the amount collected for children's tickets and *a* = the amount collected for adult tickets, then our system of equations will look like this:** [Write/show and speak.] $c + a = 800$ $7.50c + 13.50a = 7{,}680$ **First, we'll solve one of the equations for one variable. If you can follow along and solve for *c* yourself, great! Here's what we get when we solve for *c*:** [Write/show and speak.] $c + a = 800$ $c = 800 - a$ **You can substitute 800 – *a* for *c* in the other equation, then solve. Try writing this with me:** [Write/show and speak.] $7.50(800 - a) + 13.50a = 7{,}680$ $6{,}000 - 7.50a + 13.50a = 7680$ $6a = 1{,}680$ $a = 280$	Objective 1: Students discover when to use the substitution method for solving a system of equations.

From this, we know that 280 adult tickets were sold. Now, let's solve for the other variable. You can substitute 280 for *a* in either equation and solve for *c*. Follow along and write down the answer as well: [Write/show and speak.] **$c + a = 800$** **$c + (280) = 800$** **$c = 520$** **Now we can determine that 520 children's tickets and 280 adult tickets were sold on Saturday.** **What if you wanted to substitute the value for *a* in terms of *c* in the equation? What value for *a* would you use?** [pause] **That's right, you would simply solve $c + a = 800$ for *a* instead, to get $a = 800 - c$.** [Write/show and speak.] **Let's look at another problem. Listen to all the information first then get that pencil and paper ready!**	
Objective 2 Example 1 – Use substitution to solve a system with no solution. [Write/show and speak the problem.] **Seiko takes pottery classes at two different studios. At the community pottery studio, the cost is $14 per hour with the cost of clay included. At the private pottery studio, the cost is $28 per hour plus $15 for clay. Is there a number of hours for which Seiko's cost is the same at both studios?** **So now, we'll write our system of linear equations together. Let *x* equal the number of hours and *y* equal Seiko's cost.** [Write/show and speak.] **$y = 14x$** **$y = \frac{1}{2}(28x + 15)$** **Since both equations are set equal to y, what would you do first?** [Pause] **That's right, you would set up an equation using the two expressions that are equal to y.** **We can use substitution to solve one of the equations for one variable.** [Write/show and speak.] **$14x = (28x) + 15$** **$14x = 14x + 7.50$** **$0 \neq 7.50$**	Objective 2 Example 1: Students will use substitution to solve a system with no solution.

The result is not a true statement, so the system has no solution. There is no number of hours for which Seiko's cost is the same at both studios. If we look back, there are some clues that this will be a false statement – for instance, if you recall earlier lessons on slope intercept, the slopes for this problem would be the same but the y-intercepts are different. Also, the two lines made from the equations would be parallel.	
Objective 2 Example 2 – Use substitution to solve a system with infinitely many solutions. **Now lets' try a different type of problem. I'd like you to follow along and try to write out as much of these as you can. Take a look at this system and solve by using substitution.** [Teacher write/show and speak the problem.] **$-2y - x = -84.91$** **$3x + 6y = 254.73$** **First, solve the equations for one variable.** [pause] **Let's write that out now:** [Teacher write/show and speak the problem.] **$-2y - x = -84.91$** **$-x = -84.91 + 2y$** **$x = 84.91 - 2y$** **Let's use substitution now to solve the problem. Substitute $84.91 - 2y$ for x in the other equation, then solve. It will look something like this:** [Teacher write/show and speak the problem.] **$3(84.91 - 2y) + 6y = 254.73$** **$254.73 - 6y + 6y = 254.73$** **$254.73 = 254.73$** **The result is a true statement. This system has infinitely many solutions. What happened as a result of the substitution in step 2? The result was a true numerical statement with no variables. A system with an infinite number of solutions is made up of equivalent equations. We can see that this is true in this example because if you multiply the first equation by -3, both equations are $3x + 6y = 254.73$.** <u>Additional problem time permitting</u>	Objective 2 Example 2: Students will use substation to solve a system with many solutions. Additional Problem: The student will decide which equation to choose to begin to work with. In this case, the student should consider the

Let's look at another problem. [Write/show and speak the problem.] **Brandon took a 50-question exam worth a total of 160 points. There were *x* two-point questions and *y* five-point questions. How many of each type of question were on the exam?** **Based on this information, what should our system of equations look like? Well, we already have determined that *x* is the two-point questions, and *y* is the five-point questions, so it should look something like this:** [Write/show and speak.] ***x* + *y* = 50** **2*x* + 5*y* = 160** **Now let's solve for one of the variables.** [Write/show and speak.] ***y* = 50 – *x*** **Did you get the same answer? Let's continue. We can then substitute for *y*, giving the following:** [Write/show and speak.] **2*x* + 5(50 – *x*) = 160** **2*x* + 250 – 5*x* = 160** ***x* = 30** **Now we see that there are 30 two-point questions. With this information, we can solve for *y*:** **30 + *y* = 50** ***y* = 20** **Would the problem have the same solution if you first isolated *x* instead of *y*?** [pause] **That's right, it would, because the solution is the same either way.** **Let's look at a few more, and you work along with me.**	equation that has at least one variable with a coefficient of 1.
<u>Guided Practice</u> (10-12 min) [I Do] **I'll walk through one more for you. Listen to the information first, like we did with the other problems, and then grab your pencil and paper to follow along.** [write/show and speak the problem] **Amy and Mabel are taking a trip this summer. Together, they have saved $490 for their trip. Amy has saved $54 more than Mabel. How much has each person saved?** **Let's write a system of linear equations to represent the situation:** [Teacher write/show & speak.] ***a* + *m* = 490** ***a* = *m* + 54**	Students use their knowledge of solving systems of equations using the substitution method to determine if each system has one, none, or infinitely many solutions.

As you can see, *a* = the amount Amy has saved, while *m* = the amount Mabel has saved. **If we use substitution to find the value of *m*, it would look like this:** [Teacher write/show & speak.] **(*m* + 54) + *m* = 490** **2*m* + 54 = 490** **$m = 218$** **Once we have *m*, we can substitute to find the value of *a*:** ***a* = 218 + 54** ***a* = 272** **So then we see that Amy has saved $272, while Mabel has saved $218.** [We Do] **Now, here's one more for you to try mostly on your own. If you have a calculator and want to use it to check your work, that's OK.** [Write/show and speak the problem.] **A parking lot charges $7 per day to park on weekdays and $12 per day on weekends. Jamal parked his car in this lot for 6 days last week and spent a total of $52. How many weekdays and weekend days did Jamal park?** **Here I've got a system of equations where *x* = the number of weekdays and *y* = the number of weekend days. Can you fill in the blanks? Write this on your paper and try to finish it.** [Teacher write/show and then pause to let the student think of the missing parts.] ***x* + *y* = ___** **__*x* + __*y* = 52** [pause] **Did you get something like this?** [Write/show and speak.] ***x* + *y* = 6** **7*x* + 12*y* = 52** **Great! Let's move on, and solve the equation for *y* in terms of *x*. Can you fill in the blanks?** [Write/show with blanks.] ***y* = ____ - ____** [pause] **Did you get something like this?**	

y* = 6 - *x **You can rewrite the second equation by substituting your value for y in terms of x. It will look something like this:** [Write/show with blank.] **7*x* + 12(_____) = 52** **Can you complete?** [Pause, giving time for student to answer.] **It should look like this:** [Write/show and speak.] **7*x* + 12(6-*x*) = 52** **Let's solve the equation. What is the value of *x*?** [pause] **That's right! It's *x* = 4** **How can you find the value of *y*? What is the value of *y*?** [pause] **If you said you can substitute 4 for *x* in the equation *y* = 6 – *x*, or *y* = 2, then you are correct! You're doing great at this!** [You Do] **Here's one for you to try mostly on your own.** [Teacher read/show the problem.] **Krysta bought 24 notebooks and spent $104. The large notebooks cost $6 each and the small notebooks cost $2 each. How many of each type of notebook did she buy? Let's say that *l* = large student notebooks, and *s* = small student notebooks. Now, let's set up your equations.** [Pause for student to answer.] **Did you set up equations that look like this?** [Write/show.] ***l* + *s* = 24** **6*l* + 2*x* = 104** **Great! So how many of each type of notebook did she buy?** [pause] **If you got 14 large notebooks and 10 small notebooks, congratulations!** <u>Additional Problems if needed:</u> **Let's look at a scenario involving money. Follow along with me.** **Mario has $7.50 in dimes and quarters. The total number of coins is 48. How many dimes and quarters does he have?** **What information do you need that is not given in the question? That's right, we need the values of the dimes and**	

quarters! Of course, we know that a dime is 10 cents, and a quarter is 25 cents, so let's write a system of equations to model this. [pause] **$0.10d + 0.25q = 7.50$** **$d + q = 48$** **Or, you could eliminate the decimals and write it this way:** **$10d + 25q = 750$** **$d + q = 48$** **Use substitution to solve. How many dimes and quarters does Mario have?** [pause] **Did you find that Mario has 30 dimes and 18 quarters?** **You really are getting this! Keep going with some independent practice.**	
Independent Practice (1 min) **Superb work today, students! Today, we explored Solving Systems of Linear Equations using the substitution method. After this lesson, you will have a few problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, www.tn.gov/education.** [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	
Closing (1 min) **I enjoyed solving systems of linear equations using the substitution method with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	