PBS Lesson Series

Math: Grade 8, Lesson 15, Solving Systems by Elimination Method using Multiplication

Lesson Focus: Solving Systems by Elimination Method using Multiplication
Practice Focus: Students will practice solving systems of linear equations with a focus on using the elimination method with multiplication.
Objective:

- Identify common multiples of coefficients.
- Solve systems of equations using elimination with multiplication.
- Review solving systems of equations – multiple methods.

Key Vocabulary:

- Elimination, Coefficients, Multiplication Property of Equality

TN Standards: 8.EE.C.8
Teacher Materials:

- Whiteboard and Markers, Graph Paper if available or Coordinate plane board, Straight edge
- Student Practice Packet

Student Materials:

- Paper and a pencil, and a surface to write on
- Calculator not required but may be used to check calculations.
- For the student practice, a straight edge may be useful in graphing lines in the cumulative review.

Note: There are a charts and graphs that will need to be prepared ahead of time to show to students during the lessons this week. In today's lesson, the student practice worksheet includes a cumulative review of solving systems of linear equations.

Teacher Do	Student Do
<u>Opening</u> (1 min) **Hello! Welcome to Week 3 of Tennessee's At Home Learning Series for math! Today's lesson is for all our 8th graders out there, though all students are welcome to tune in. This lesson is the fifteenth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. If you don't already have the student packet for this lesson, you can find it online at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.** **Today we will be continuing our learning about systems of linear equations, and we will be working on estimating**	Students get materials ready for the lesson.

solutions by graphing! Before we get started, to participate fully in our lesson today, you will need:

- **Paper and a pencil, and a surface to write on**
- **A calculator is not required but may be used to check calculations.**
- **Optional but helpful materials would be a straight edge and graph paper for the student practice worksheet today which includes a cumulative review of our lessons on solving systems of linear equations this week.**

Ok, let's begin!

<u>Intro</u> (2 min)
We are finishing up Week 3 of our 8th grade mathematics learning series, and we are still working with equations and expressions this week. Let's warm with a little mathematical practice.

In Lesson 14, we looked at this chart about maximum heart rates based on a person's age. [Teacher will show the chart.]

Heart Rates (beats per minute)	
Maximum Heart Rate (MHR)	**220 - age**
Vigorous Intensity Exercise	**70-85% of MHR**
Moderate Intensity Exercise	**50-70% of MHR**

Remember that x represents the age of a person, and y represents the maximum heart rate. Here's what we wrote in Lesson 11 [Write on whiteboard.] **y = 220 – x.**

Now, let's write and simplify an expression that gives the heart rate that is 70% of the maximum heart rate for a person who is *x* years old. Think about how we would modify the equation. [Pause]
If we want to represent 70% of the maximum heart rate, we can write the equation this way: [Teacher will write/show and speak.]
y = 0.70(200 – x)

How would we change this function to solve for the person's age, *x*? Think about that for a minute and see what you can come up with. [pause]
See if you can follow my steps. [Teacher write/show and speak.]

$$y = 0.70(220 - x)$$
$$y = 154 - 0.70x$$

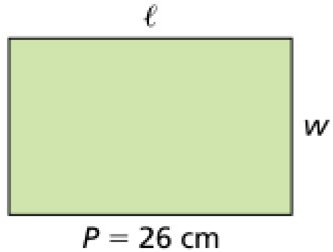
Students listen to the problem, consider what the problem is asking, and determines possible solution strategies.

$$y - 154 = 154 - 154 - 0.70x$$ $$y - 154 = -0.70x$$ $$\frac{y - 154.}{-0.70} = \frac{-0.70x}{-0.70}$$ $$\frac{y}{-0.70} + 220 = x$$ **We could clean this up a little bit, but for now, we are good to go. This equation now gives us a person's age, *x*, as a function of the maximum heart rate, *y*. Give it a try! If a person's maximum heart rate is 121.1, what is their age?** [Pause] **Did you get 47? Great warm-up! Now, let's let started with today's focus.**	
<u>Teacher Model</u> (10-12 min) Objective 1: Example 1 – Identify Common Multiples **In Lesson 14, we started looking at ways to solving systems of linear equations by the elimination method. We chose one variable to eliminate. We needed the variable to have opposite coefficients, and then we could add the equations together to solve them in one variable. That's the same thing we need today.** **So, let's first just practice finding some common multiples to help us when a system of linear equations looks like this: You can grab your paper and pencil and write the system along with me.** [Teacher writes problem] $$2x - 3y = 4$$ $$-3x + 5y = -3$$ **What do you notice about the coefficients of the *x* terms and the *y* terms?** [Pause] **So, you probably noticed that 2x and -3x could not be eliminated in this system just as they are by either addition or subtraction because they do not have the same or opposite coefficients. So, how could we use the multiplication property of equality to transform these equations so that the coefficients of *x* have the same or opposite coefficients?** [Pause] **Remember that the multiplication property of equality tells us that if we multiply every term in an equation by the same number, the resulting equation is equivalent to the original.**	Objective 1: Students will review common multiples and identify common multiples of values of coefficients in systems of equations.

Let's look at the coefficients of the *x* term. In this case, we want to think about what is a common multiple of 2 and 3. [Point to 2 and 3 in the equations.] **There are several to choose from that would work. See if you can come up with at least two common multiples of 2 and 3. Write them down on your paper.** [Pause] **Did you come up with any of these numbers as common multiples?** [Teacher will write/show and speak.] **6, 12, 18** **Great!** **Now, let's look at the coefficients of the *y* terms which are -3 and 5** [Point to these in the equations.] **Think for a minute about finding the common multiple of 3 and 5, and try to come up with at least two different ones and make a list on your paper.**[Pause] **Did you come up with at least two of these?** [Teacher will write/show and speak.] **15, 30, 45** **Great!** **Let's move on to using this skill to help us solving systems of linear equations using multiplication.** Objective 2: Example 1: Solving Systems of Linear Equations by Elimination with Multiplying **Let's go back to our system of equations that we looked at to start.** [Point back at system or rewrite/show/speak.] $\mathbf{2x - 3y = 4}$ $\mathbf{-3x + 5y = -3}$ **Remember that we can choose to eliminate either the *x* terms or the *y* terms, but we will need to transform these equations by multiplication so that one set of the terms have opposite coefficients.** **For this example, I will choose to eliminate *x*. I know from before that 6 is a common multiple of 2 and 3. So, in this case, I know that I can multiply 2 by 3 and get 6, and 3 by 2 to get 6. Let's do that.** $\mathbf{3(2x - 3y = 4)}$ $\mathbf{2(-3x + 5y = -3)}$ **When we multiply each equation, don't forget to multiply every term to preserve the equivalency of the transformed equations to the originals.** [Write/speak the transformed equations.] $\mathbf{6x - 9y = 12}$ $\mathbf{-6x + 10y = -6}$	Objective 2: Students solve systems of linear equations by elimination with multiplying to create a common multiple for the elimination method.

Now, we are ready to simply add the equations together like we did in Lesson 14 since the coefficients of one of the terms, in this case the *x* term, are opposites. $$\begin{array}{c} \cancel{6x} - 9y = 12 \\ \underline{-\cancel{6x} + 10y = -6} \\ y = 6 \end{array}$$ **Now, that the *x* terms are eliminated, we come up with y = 6 meaning that the value of the y of the solution ordered pair is 6. Now, to find the value of the *x* in the solution ordered pair, simply choose one of the equations – whether it is one of the original equations** [Point at the original set.] **or one of the transformed equivalent equations** [Point at the transformed set.] **I will choose the first equation in the original set and substitute the value of *y* in the equation to find *x*. Follow along with me.** [Teacher will write/show and speak.] $$\begin{array}{c} 2x - 3y = 4) \\ 2x - 3(6) = 4 \\ 2x - 18 = 4 \\ 2x - 18 + 18 = 4 + 18 \\ 2x = 22 \\ \frac{2x}{2} = \frac{22}{2} \\ x = 11 \end{array}$$ **So, we can say that the solution of this system of equations if the ordered pair (11, 6).** Objective 2: Example 2: Solving Systems of Linear Equations by Elimination with Multiplying **Let's try another one. Let's look at this system. You write it along with me.** [Teacher will write/show and speak.] $$\begin{array}{c} 3x - 5y = -9 \\ x + 2y = 8 \end{array}$$ **Looking at the *x* terms and *y* terms, what are the common multiples of each set?** [Pause] **Right. In this case, the coefficients of the *x* terms are 1 and 3, and some common multiples are just 3, 6, or 9. The coefficients of the *y* terms are -5 and 2, and some common multiples would be 10 or 20.** **Remember, you can chose which variable you want to eliminate. I'm going to choose to eliminate the *y* term in this case because they are already opposite signs.** **I'll use 10 as my common multiple. Follow along with me.** [Teacher will write/show and speak.] $$3x - 5y = -9$$	Objective 2 (cont.): Students solve systems of linear equations by elimination with multiplying to create a common multiple for the elimination method.

$\boldsymbol{x + 2y = 8}$ $\boldsymbol{2(3x - 5y = -9)}$ $\boldsymbol{5(x + 2y = 8)}$ $\boldsymbol{6x - 10y = -18}$ $\boldsymbol{5x + 10y = 40}$ $\boldsymbol{6x - \cancel{10y} = -18}$ $\boldsymbol{\underline{5x + \cancel{10y} = 40}}$ $\boldsymbol{x = 22}$ **Since the *x* terms in the solution ordered pair is equal to 22, we just choose an equation to substitute that value back in to find the value of the *y* terms in the solution ordered pair. I'll choose the second original equation this time.** [Point at the x + 2y = 8 equation.] **Remember that you can choose any of the equations in this system – either the original system or the transformed equivalent system.** [Teacher will write/show and speak.] $\boldsymbol{x + 2y = 8}$ **Substitute in 22 for the x** [Write/show and speak the steps.] $\boldsymbol{22 + 2y = 8}$ $\boldsymbol{22 - 22 + 2y = 8 - 22}$ $\boldsymbol{2y = -14}$ $\boldsymbol{\frac{2y}{2} = \frac{-14}{2}}$ $\boldsymbol{y = -7}$ **So, we can say that the solution of this system of equations if the ordered pair (22, -7).** **Let's do another example with a little context this time.** Objective 2: Example 3: Solving Systems of Linear Equations by Elimination with Multiplying [Read or show the problem with the graphic representation.] **The difference of the length and width of the rectangle is 3 centimeters. The perimeter of the rectangle is 26 centimeters. What are the dimensions (length and width) or the rectangle?** [Pause] **Go ahead and sketch the picture of the rectangle and write down the perimeter.**	Objective 2 (cont.): Students solve systems of linear equations by elimination with multiplying to create a common multiple for the elimination method.



We will recall that the perimeter of a rectangle is found by the following formula where *w* represents the width and *l* represents the length. [Teacher will write/show and speak.]

$$2l + 2w = P$$

We know from the problem that the perimeter is equal to 26. So, we can write the equation this way: [Teacher will write/show and speak.]

$$2l + 2w = 26$$

To complete the system, we know from the problem statement that the difference between the length and width is 3 centimeters. We can write the equation this way: [Teacher will write/show and speak.]

$$l - w = 3$$

So, if we want to solve the system by elimination, let's set this up: [Teacher will write/show and speak.]

$$2l + 2w = 26$$
$$l - w = 3$$

We can choose a variable to eliminate. In this case, I think I will go with length or *l*. [Point to the terms.]

The coefficients of the *l* terms are 2 and 1. I need to think about a common multiple, and in this case, it's just 2. However, I also need one of the coefficients to be opposites. Here's how I will handle that. Follow along with me. [Write/show and speak.]

$$2l + 2w = 26$$
$$l - w = 3$$

If I want the coefficient of the *l* in the second equation to be -2, I will just multiply the second equation by -2. Like this.

$$2l + 2w = 26$$
$$-2(l - w = 3)$$

The transformed equations look like this:

$$2l + 2w = 26$$
$$-2l + 2w = -6$$

Now, we are ready to add the equations to start solving for the solution ordered pair.

$$2l + 2w = 26$$
$$\underline{-2l + 2w = -6}$$
$$4w = 20$$
$$\frac{4w}{4} = \frac{20}{4}$$

$$\boldsymbol{w = 5}$$ **So, the width of the rectangle is 5 centimeters. Let's choose an equation to substitute in and find the length. I'll choose the first original equation.** [Point to the 2l + 2w = 26.] **Follow along with me one more time.** [Teacher will write/show and speak.] $$\boldsymbol{2l + 2w = 26}$$ $$\boldsymbol{2l + 2(5) = 26}$$ $$\boldsymbol{2l + 10 = 26}$$ $$\boldsymbol{2l + 10 - 10 = 26 - 10}$$ $$\boldsymbol{2l = 16}$$ $$\boldsymbol{\frac{2l}{2} = \frac{16}{2}}$$ $$\boldsymbol{l = 8}$$ **We know that the length of the rectangle is 8 centimeters.** **Since Lesson 15 is an extension of the work you did in Lesson 14, think about how you are feeling about your ability to choose a variable to eliminate, multiply the equations using the multiplication property of equality in order to create opposite coefficients in one of the terms to allow you to continue solving the system of equations by elimination.** [Pause] Objective 3: Example 1: Review solving systems of equations – multiple methods **This week, we have looked at several strategies or methods for solving systems of equations including inspection, graphing, substitution, and elimination.** **Sometimes you will be told which strategy or method to use, but other times, you will be able to choose which method you want to choose. A fluency goal is that you can quickly make a decision about which strategy in a given situation. So, let's practice this for a minute.** **Let's look at a few different systems of linear equations and make a decision about which strategy or strategies might be more efficient than another. Here's our first system. Remember to write this one along with me.** [Teacher will write/show and speak.] $$\boldsymbol{y = 5x + 2}$$ $$\boldsymbol{y = -3x - 1}$$ **You can use any one of the strategies we've looked at this week including inspection, graphing, substitution, and elimination. However, the structure of the system might give us some guidance about which strategy might be a good**	Objective 3: Students review and identify potential strategies to solve systems of equations.

choice. Which ones do you think would work well here?
[Pause]
We notice that these equations are in slope-intercept form. That might tell us that graphing could work, but we also know that substitution would be a good option here as well.

Let's look at another system and think about what strategy or strategies might be more efficient than another.
[Teacher will write/show and speak, then pause.]

$$\begin{gathered} \boldsymbol{y - x = 28} \\ \boldsymbol{y + x = 156} \end{gathered}$$

In this system, we notice that the equations are not in slope-intercept form so graphing might not be our first choice. We Substitution could work because the equations would be easy to transform into a slope intercept form. However, in this situation, you might choose to use elimination because you already have opposite coefficients on the *x* terms.

Remember that when solving systems of equations, you have two other options for solutions. You might have NO solutions or INFINITE solutions. In Lesson 13, you explored what that looks like when solving by substitution. We didn't demonstrate any systems like that today, but if we did, do you remember which one represents a system with NO solutions and which one represents a system with INFINITE solutions? Check these systems out that have been solved by elimination and substitution [Teacher will write/show and speak.]

System A:

$$\begin{gathered} \underline{\boldsymbol{4x + \cancel{y} = 3}} \\ \underline{\boldsymbol{-4x - \cancel{y} = \ -8}} \\ \boldsymbol{0 = \ -5} \end{gathered}$$

System B:

$$\begin{gathered} \boldsymbol{8\left(\frac{1}{4}x - 2\right) - 2x = \ -16} \\ \boldsymbol{2x - 16 - 2x = -16} \\ \boldsymbol{-16 = -16} \end{gathered}$$

Which system shows us that there are NO solutions? [pause] **Right! System A shows a NO solution statement since 0 = -5 is a false statement. Now, what about System B?** [pause] **Right again! System B shows what an INFINITE solution statement might look like since -16 = -16 is a true statement. Just keep this possibility in in mind as you work through your problems.**

<u>Guided Practice</u> (10-12 min) [I Do] **Now, I'll walk through one more system of equations today. Now, let's start working through one together.** **Let's look at this system of equations.** [Teacher will write/show and speak.] $\mathbf{3x + 6y = 18}$ $\mathbf{6x - 2y = 22}$ **Let's think first about which strategy might be more efficient. In this case, it looks like elimination will work well. We just need to pick a variable to eliminate and transform the equations by multiplication to create opposite coefficients. Because *y* is already opposite signs, I'll choose to eliminate that variable.** **The coefficients are 6 and 2** [Point at 6 in equation 1 and 2 in equation 2.]. **So, what common multiple do we have of 6 and 2?** [Pause] **Right! 6 is a common multiple. So, we only have to multiply the second equation to create the opposite coefficients for *y*. Let's walk through this together.** [Teacher will write/show and speak.] $\mathbf{3x + 6y = 18}$ $\mathbf{6x - 2y = 22}$ **Multiply the second equation by 3.** $\mathbf{3x + 6y = 18}$ $\mathbf{3(6x - 2y = 22)}$ **You will get this system:** $\mathbf{3x + 6y = 18}$ $\mathbf{18x - 6y = 66}$ **Now, eliminate the *y* by adding the equations together to solve:** $\mathbf{3x + \cancel{6y} = 18}$ $\underline{\mathbf{18x - \cancel{6y} = 66}}$ $\mathbf{21x = 84}$ $\mathbf{\frac{21x}{21} = \frac{84}{21}}$ $\mathbf{x = 4}$ **We are ready to solve for y using any one of the equations. I'll choose the first original equation. Follow along.** [Teacher will write/show and speak.] $\mathbf{3x + 6y = 18}$ **Substitute in 4 for x** $\mathbf{3(4) + 6y = 18}$ $\mathbf{12 + 6y = 18}$ $\mathbf{12 - 12 + 6y = 18 - 12}$	Students use their knowledge of solving systems of linear equations by elimination (and by other strategies if time allows) to solve the systems.

$$6y = 6$$
$$\frac{6y}{6} = \frac{6}{6}$$
$$y = 1$$

So, we know the solution to this system of equations is the ordered pair (4, 1). Recall that this means that this ordered pair makes BOTH the system equations true.

[We Do]
Okay. Let's investigate this system. What strategy do you think will work best to solve this system? [Teacher will write/show and speak, then pause.]

$$y - x = 28$$
$$y + x = 156$$

So, what strategy did you choose? [Pause]
If you are like me, then you chose elimination, but I know we don't need to multiply anything because the *x* terms already have opposite coefficients.

So, let's get started with this part. Go ahead and start solving. I'll give you a minute to get started and see if you can get the value of one of the ordered pairs. [Pause]

$$y - x = 28$$
$$y + x = 156$$

If you solved it like I did, you just added the equations and ended up with the value of *y* this way.

$$2y = 184$$
$$\frac{2y}{2} = \frac{184}{2}$$
$$y = 92$$

Now, you choose an equation and see if you can get the value of *x* in this system. I'll give you a minute to work. [Pause]
If you solved it like I did, you chose the second equation to substitute in. Just remember that because we are finding the solution to the system, you can choose either one of the equations to solve for the missing value. Here's what I did. [Teacher will write/show and speak.]

$$y + x = 156$$
$$92 + x = 156$$
$$92 - 92 + x = 156 - 92$$
$$x = 64$$

Did you get the same solution set? You can write it this way as an ordered pair: (64, 92)

[You Do]

Terrific! Now, here's one more for you to try mostly on your own. You can choose a strategy that seems to make the most sense for the system. [Teacher will write/show and speak.] $$3x + 2y = 17$$ $$6x - 2y = 28$$ [Pause – give students time to work.] **Did you choose elimination? I did because the coefficients of the *y* terms are already opposites. Once I added the equations together, I ended up with this to solve for *x*.** [Teacher will write/show and speak.] $9x = 45$ **And I calculated that *x* has a value of 5. Then, I chose the first equation to substitute that value you in. What did you get for *y*?** [Pause] **This is the equation I got once I substituted in 5.** [Teacher will write/show and speak.] $$15 + 2y = 17$$ **Then I solved it and discovered that y = 1. How about you?** [Pause] **Good work! Our solution to this system is the ordered pair (5, 1).** Additional Practice if needed: [These examples will give a substitution and a graphing option.] **Let's review the substitution method of solving a system of equations. Start by looking at this system of linear equations and determining if one of the equations is simpler to transform than another.** [Teacher will write/show and speak.] $$3x + 4y = 33$$ $$2x + y = 17$$ **Based on our work in Lesson 14 and Lesson 15 today, I think most of you would say that elimination would be the better strategy to use, and I would agree. However, we will need to practice all strategies. So, let's think about the substitution method.** **I'll start by solving the second equation for *y*. Like this:** [Teacher will write/show and speak.] $$2x + y = 17$$ $$2x - 2x + y = 17 - 2x$$ $$y = 17 - 2x$$	

Now, we remember to replace the *y* term in the first equation with the expression from the second equation. Take a minute and see if you can get that set up. [Pause]
This is how I started. Follow along with me: [Teacher will write/show and speak.]

$$\mathbf{3x + 4y = 33}$$
$$\mathbf{3x + 4(17 - 2x) = 33}$$
$$\mathbf{3x + 68 - 8x = 33}$$
$$\mathbf{-5x + 68 = 33}$$
$$\mathbf{-5x + 68 - 68 = 33 - 68}$$
$$\mathbf{-5x = -35}$$
$$\mathbf{\frac{-5x}{-5} = \frac{-35}{-5}}$$
$$\mathbf{x = 7}$$

Now, take the 7 and substitute that value back in the transformed equation to find *y*. Follow along: [Teacher will write/show and speak.]

$$\mathbf{y = 17 - 2x}$$
$$\mathbf{y = 17 - 2(7)}$$
$$\mathbf{y = 17 - 14}$$
$$\mathbf{y = 3}$$

So, in this case, the solution to this system is the ordered pair (7, 3).

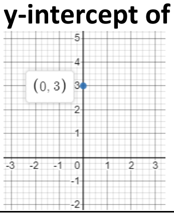
Another system of linear equations looks like this: [Teacher will write/show and speak.]

$$\mathbf{y = -4x + 3}$$
$$\mathbf{y = -4x + 4}$$

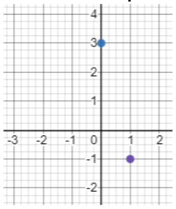
Now, what strategy will work best for this system? [Pause]
If you said graphing, then you chose what I chose. Let's practice graphing this system.

For each equation identify the y-intercept and the slope. [Pause]
Did you get a y-intercept of 3 and a slope of -4 in the first equation? How about the second equation? [Pause]
Right! The y-intercept if 4 and a slope of -4 for the second equation.

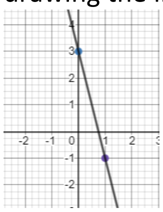
Let's graph these on a coordinate plane. Start by plotting the y-intercept of the first equation at (0,3)



Then, use the slope of -4 to plot the next point. Move down four and one to the right from the y-intercept. [Model the shifts and plot the point.]

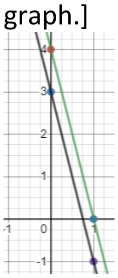


Use a straight edge if you have one and draw the line. Make sure to extend the line beyond the two points. [Model drawing the line.]



Now, you try plotting the second equation on the same coordinate plane. I'll give you a minute to try. [Pause]

Here's what I got. Does yours look like mine? [Show the graph.]



These are parallel lines – so remember that this means that there is NO solution to this system of equations.

How do you feel about everything you've learned this week? Don't forget you can go back and review any lesson for additional practice.

<u>Independent Practice</u> (1 min)

Superb work today, students! Today, we explored Solving Systems of Linear Equations by Elimination with Multiplying and some cumulative review from the week. After this lesson, you will have a few problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, <u>www.tn.gov/education</u>.
[Teacher shows student practice page under document camera or camera zooms in on student practice page.]
Good luck and do your best!

Closing (1 min) **I enjoyed finishing our week exploring strategies solve systems of linear equations with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	