PBS Lesson Series

Math: Grade 8, Lesson 11, Analyze and Solve Systems of Linear Equations

Lesson Focus: Estimate Solutions by Inspection
Practice Focus: Students will focus on understanding that linear equations can have no solution, one solution, or infinitely many solutions. Students will relate the number of solutions to a linear system to the slopes and y-intercepts of the graphed lines and see that a solution is any ordered pair that makes all equations true.
Objective:

- Examine the graphs of a linear system of equations to determine the number of solutions of the system.
- Compare the equations in a linear system to determine the number of solutions of the system.

Key Vocabulary:

- Solution of a system of linear equations
- System of linear equations

TN Standards: 8.EE.C.8a
Teacher Materials:

- Whiteboard and Markers, Graph Paper if available
- Student Practice Packet

Student Materials:

- Paper and a pencil, and a surface to write on
- Calculator not required but may be used to check calculations.
- Optional but helpful: Graph Paper

Note: There are a charts and graphs that will need to be prepared ahead of time to show to students during the lessons this week.

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Week 3 of Tennessee's At Home Learning Series for math! Today's lesson is for all our 8th graders out there, though all students are welcome to tune in. This lesson is the eleventh in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. If you don't already have the student packet for this lesson, you can find it online at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.**	Students get materials ready for the lesson.

Today we will be learning about systems of linear equations, and we will be working on estimating solutions by inspection! Before we get started, to participate fully in our lesson today, you will need:

- **Paper and a pencil, and a surface to write on**
- **A calculator is not required but may be used to check calculations.**
- **Optional but helpful material would be graph paper**

Ok, let's begin!

Intro (2 min)

We are starting Week 3 of our 8th grade mathematics learning series, and we are moving back into equations and expressions this week.

Have you ever seen a person wearing a fitness tracker? It usually looks like a watch, but it can tell the wearer a lot about fitness and activity. One of the things these trackers are very good at is counting how many steps you've taken, or how many flights of stairs you've climbed. They do this by using devices called accelerometers to count steps or stairs, and they also use a Global Positioning System, or GPS, to help determine how far you've gone in miles or kilometers. Some trackers even monitor heart rate by shining a light into your blood vessels and calculating the rate based on how the light is reflected back.

That's some sophisticated technology! You can also use this data to find out other things. Take a look at the information in this chart [show chart on whiteboard]**. Let's write an equation that gives maximum heart rate, y, for a person who is x years old.**

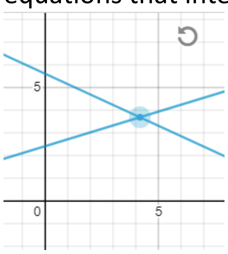
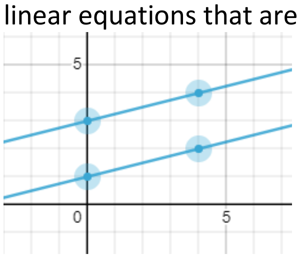
Heart Rates (beats per minute)	
Maximum Heart Rate (MHR)	**220 - age**
Vigorous Intensity Exercise	**70-85% of MHR**
Moderate Intensity Exercise	**50-70% of MHR**

Remember that x represents the age of a person, and y represents the maximum heart rate. Here's what you should have: [write on whiteboard] **$y = 220 - x$**

What is the meaning of the slope and y-intercept in this context? [pause]

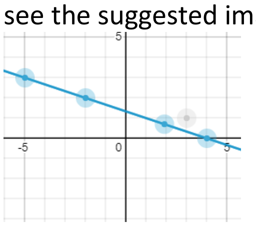
What did you think about? [pause] **I noticed that 220 is the y-intercept, and this seems to tell me that this is the**

Students listen to the problem, consider what the problem is asking, and determines possible solution strategies.

maximum heart rate for an infant. Also, the slope of -1 tells me that the maximum heart rate decreases as a person gets older. We will use this scenario each day as we get started this week. **Now, are you ready to dive into some systems of equations?** [Pause] **OK, let's get started!**	
<u>Teacher Model</u> (10-12 min) Objective 1: Example 1 - Examine the graphs of a linear system of equations to determine the number of solutions of the system. **We will look at the graphs of three different systems of linear equations. Remember that a system of linear equations is just two or more linear equations that we are comparing. A graph of a system of linear equations is just two or more linear equations displayed on the same coordinate plane.** **Here's the first one.** [Teacher draws or shows a set of linear equations that intersect at one point – see image suggestion.]  **What do you notice about this system? How are these lines related?** [pause] **Do these lines cross each other?** [pause] **Exactly – they cross one time on the graph.** **Now, what do you notice about the slope of each line?** [pause] **Right! The slopes appear to be different from each other.** **Let's look at the second one.** [Teacher draw or shows a set of linear equations that are parallel – see image suggestion.]  **What do you notice about this system? How are these lines related?** [pause] **Do these lines cross each other?** [pause]	Objective 1: Students will identify characteristics of systems of linear equations that have ONE solution, NO solutions, or INFINITE solutions.

Right! They don't cross, and they don't look like they would even if we extended the lines a little further. Do you remember what type of lines these are called? [pause]
If you said "parallel lines", then you're right!
Now, what do you notice about the slope of each line? [pause]
Right! Since the lines appear to be parallel, they have the same slope, but what is different about these? [pause]
If you started thinking about different y-intercepts, then you are right again!

Let's look at a third scenario. [Using two different colored markers if possible, draw two lines one on top of the other – see the suggested image.]



Now, this is a little more challenging visually, but what do you notice about these two lines? [pause]
Right! When I showed you both lines, they are actually the same line on the graph. So, what does that mean about their slope? [pause]
True! If the lines are the same, they must have the same slope. And now the y-intercept? [pause]
Exactly! Those are the same as well!

Each of these situations represents one of the possible outcomes for a system of linear equations. In the first situation [Teacher point at or show the two intersecting line drawing.], **we see that the lines cross at one point and one point only. We say that this system of linear equations has ONE solution. The solution to a system of linear equations is any ordered pair that makes both the equations true. In, this case, the intersection of the lines is the point with the ordered pair that makes both equations true.** [Point at the point of intersection.]

In the second situation [Teacher point at or show the two parallel lines drawing.], **we see that this system of linear equations are parallel and never intersect. We say that this system of linear equation has NO solutions. There are no ordered pairs that make both the equations true.**

Finally, let's look at the third situation [Teacher points at or shows the two lines that are identical drawing]**, we see that since these lines are actually the same line, this must mean that EVERY point is the same, and therefore, we say that this system of linear equations has INFINITE solutions. There are an infinite number of ordered pairs that make both equations true.** **Remember these key points. You can write these down to help you remember later.** [Teacher write or show these.] • **If the equations in a system of linear equations have different slopes, they will have one solution.** • **If the equations in a system of linear equations have the same slope but different y-intercepts, they will have no solutions.** • **If the equations in a system of linear equations are the same line, they will have infinite solutions.**	
Objective 2: Example 1 - Compare the equations in a linear system to determine the number of solutions of the system. **Let's think about this in context. Harrison and Mia each buy *x* number of comic books from a store that sells them for $5 each. Harrison also buys an action figure for $15, and Mia buys a different action figure for $12. Could they each spend the same amount of money, *y*, and buy the same number of comic books?** **Let's first remember what we know about writing linear equations from descriptions like this.** **We know that *x* is the number of comic books bought, and *y* is the total amount spent. We know that Harrison spent $15 while Mia spent $12 on an action figure. So, we can write the equations like this:** [Teacher writes/shows] **Mia: $y = 5x + 12$** **Harrison: $y = 5x + 15$** **Now that we have the system of equations written, let's inspect this system by looking at the slopes and y-intercepts. What is the slope of each equation and are they the same or different from each other?** [pause] **If you said the slope was 5** [Teacher points at 5 on the written equations.] **and that the slope is the same in both, then you are correct! Now, what does that mean about the number of solutions to this system?** [pause]	Objective 2: Students will think about the situation in context, write the equations that model the scenario, and transform the equations to slope-intercept form. Once in slope-intercept form, students can use inspection of the slope and y-intercept to determine if there is a single solution, no solution, or infinite solutions in context.

This means that either both equations are parallel and have no solutions or they are the same line and have infinite solutions. How could we tell? [pause] **Exactly! If we check the y-intercept, then we will know if these are the same line or parallel lines.** [Point to the y-intercepts 12 & 15 in the equation.] **We see that these are different, so these equations are parallel. That means from our first definitions that this system has NO solutions, but what does that mean in context?** [pause] **In this situation, it means that Harrison and Mia cannot purchase the same amount of comic books AND spend the same total amount.**	
Objective 2: Example 2 - Compare the equations in a linear system to determine the number of solutions of the system. **Let's look at a third example. Here is a system of equations that represents the amount of money that two people spent at two different grocery stores to buy an amount of cheese, *x*, and an amount of tomatoes, *y*.** [Teacher write/show & speak.] **$6x + 2y = \$12$ where the cheese was \$6 a pound and tomatoes were \$2 a pound** **$12x + 4y = \$24$ where the cheese was \$12 a pound and the tomatoes were \$4 a pound.** **Let's see if we can determine by inspection how many solutions there are to this system.** **So, by inspection, we need to look for the slope first. However, you will notice that these equations are not in slope-intercept form. We need to transform them both into $y = mx + b$.** **Let's transform the first equation this way** [Teacher write/show and speak.] $6x + 2y = 12$ $6x - 6x + 2y = 12 - 6x$ $2y = 12 - 6x$ $\frac{2y}{2} = \frac{12}{2} - \frac{6x}{2}$ $y = 6 - 3x$ $y = -3x + 6$ **If we do the same thing and transform the second equation, $12x + 4y = 24$, we get $y = -3x + 6$**	Objective 2 (cont): Students will think about the situation in context, write the equations that model the scenario, and transform the equations to slope-intercept form. Once in slope-intercept form, students can use inspection of the slope and y-intercept to determine if there is a single solution, no solution, or infinite solutions in context. Students will also think about the common misconception that just because the lines have the same slope that they are parallel without checking the y-intercept to confirm or deny.

What do you notice? [pause] **Right! Both equations represent the same line. The slope is the same at -3** [Point to -3 on the equations.] **and the y-intercept is the same at 6** [Point to 6 on the equations.]. **So, let's think back. What does that tell us about the number of solutions to this system?** [pause] **If you first thought that because the slopes are the same that the lines were parallel and therefore, NO solutions, you were on the right track. But remember, we have to check the y-intercept as well is the slope is the same. In this case, the y-intercepts are also the same.** **So, if you said that that the system has INFINITE SOLUTIONS, then you are right!** **Let's look at a few more, and you work along with me.**	
Guided Practice (10-12 min) [I Do] **I'll walk through one more for you. Let's just look at a systems of equations.** [Teacher write/show & speak.] **y = 3x - 4** **5y – 15x – 20 = 0** **The first thing we notice is that the second equation is not in slope intercept form, and we need to transform it.** **Let's walk through that process.** [Teacher write/show and speak each step.] $5y - 15x - 20 = 0$ $5y - 15x + 15x - 20 + 20 = 0 + 15x + 20$ $5y = 15x + 20$ $\frac{5y}{5} = \frac{15x}{5} + \frac{20}{5}$ $y = 3x + 4$ **So, we now see that the second equation, once it has been transformed, has the same slope as the first equation, but the y-intercepts are different.** [point at the 4 and -4 in the respective equations] **This tell us that these equations represent parallel lines and therefore, there are NO solutions to this system.** **Now, let's start working through one together.** [We Do] **Let's look at this system of equations.** **How many solutions does this system have?** [pause] [Teacher write/show this system.]	Students use their knowledge of transforming equations to slope intercept form to inspect the system of equations to determine if each system has one, none, or infinitely many solutions.

-5y = -9x + 10
-4y = 2x + 4

So, what do we notice about this system? [pause] **Are we ready to inspect the equations by looking at the slope and y-intercept? What do you think?** [pause]
If you said, we need to transform these equations to slope-intercept form, then you are correct!
I'll walk through the first one, and then you will do the second one.

[Teacher write/show and speak.]
-5y = -9x + 10
Divide both sides by the coefficient of y or -5
$$\frac{-5y}{-5} = \frac{-9x}{-5} + \frac{10}{-5}$$
Simplify
$$y = \frac{9}{5}x - 2$$
Now, you simplify the next one [Teacher write/show.]
-4y = 2x + 4
[pause, give a little extra time]
You should have come up with something like this:
[Teacher write/show and speak.]
$$Y = \frac{-2}{4}x - 1$$
$$\text{Or } y = \frac{-1}{2}x - 1$$

Now, look at the slopes of each of the transformed equations. Are they the same or different? [pause]
Right! They are definitely different! So, what does that tell us about this system? [pause]
Yes! This system has one solution.

[You Do]
Now, here's one more for you to try mostly on your own. How many solutions does this system have? Think about how you know.

Here are the equations in the system [Teacher write/show and speak.]
-5y = -8x + 4
15y = 24x -12
[Pause giving time for students to transform and compare the equations.]
How did that go? Did you get these equations once you transformed them?

$y = \frac{8}{5}x - \frac{4}{5}$ $y = \frac{8}{5}x - \frac{4}{5}$ **And how many solutions? How do you know?** [pause] **Right! This case has INFINITE solutions because the equations have the same slope and same y-intercept or represent the same line.** **You really are getting this! Keep going with some independent practice.** Additional Problems if needed: **Let's try this system of equation.** [Teacher write/show & speak.] **2x – 9y = -5** **4x – 6y = 2** **Are the equations in a form where we can inspect or look at the equations to find the slope and y-intercept?** [pause] **If you said "no, they need to be transformed to slope-intercept or y = mx + b form", then you are right!** **Let's walk through each transformation. You can solve it along with me.** **Let's transform the first equation:** [Teacher write/show & speak each step.] $2x - 9y = -5$ $2x - 2x - 9y = -5 - 2x$ $-9y = -5 - 2x$ $\frac{-9y}{-9} = \frac{-5}{-9} - \frac{2x}{-9}$ $y = \frac{5}{9} + \frac{2}{9}x$ $y = \frac{2}{9}x + \frac{5}{9}$ **Now, let's transform the second equation:** [Teacher write/show & speak each step.] $4x - 6y = 2$ $4x - 4x - 6y = 2 - 4x$ $-6y = 2 - 4x$ $\frac{-6y}{-6} = \frac{2}{-6} - \frac{4x}{-6}$ $y = -\frac{1}{3} + \frac{4}{6}x$	

Don't forget to simplify any of the fractions that can be simplified.

$$y = \frac{2}{3}x - \frac{1}{3}$$

Now, let's look at both equations in slope-intercept form. What is the slope of the first equation?[pause]
If you said 2/9, you are correct. Now, what is the slope of the second equation? [pause]
If you said 2/3, you are also correct! What does this tell us about the system since the slopes are different? [pause]
Right! Because the equations have different slopes, the lines will intersect in one location. Therefore, they have ONE solution. Great work!

Let's look at this another way. If you have one equation in a system as y = -3x + 7 [Teacher write/show & speak that equation.]**, what would be the number of solutions in each of the following scenarios?**
[Teacher write/show & speak]
Equation A: y = -3x + 7

In this situation, both equations have the same slope [point to -3] **and y-intercept** [point to 7]. **Therefore, they represent the same line, and we know that this means they have INFINITE solutions.**

Here's Equation B: y = 3x + 5
In this situation, if we compare it back to the original equation, the slopes are different at -3 [point to -3] **and 3** [point to 3]. **Therefore, we know that these represent two lines that will intersect at one point. Therefore, they have ONE solution.**

Here's a last Equation C: y = -3x + 5
In this situation, we notice that the slopes are the same at -3 [point to -3 in each equation], **but we also look at the y-intercept. These are different at 7** [point at 7] **and 5** [point at 5]. **Therefore, these are parallel lines with the same slope but different y-intercepts. In this case, these lines will never intersect and therefore have NO solutions.**

Independent Practice (1 min) **Superb work today, students! Today, we explored Estimating Solutions to Systems of Linear Equations by Inspection. I hope you are making connections between slope and y-intercepts and solutions of systems of equations. After this lesson, you will have a few problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, www.tn.gov/education.** [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	
Closing (1 min) **I enjoyed inspecting systems of linear equations with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	