PBS Lesson Series

Math: Grade 8, Lesson 12, Solving Systems by Graphing

Lesson Focus: Solving Systems by Graphing
Practice Focus: Students will write and graph systems of linear equations to determine the number of solutions.
Objective:

- Examine the graph of a system of linear equations to determine the solution where the solution to linear system is the point or points at which the lines intersect.
- Graph systems of linear equations on a coordinate plane and examine the resulting graph to determine the solution.

Key Vocabulary:

- System of linear equations, Slope, Y-intercept, Parallel, One solution, No solutions, Infinitely many solutions

TN Standards: 8.EE.C.8b
Teacher Materials:

- Whiteboard and Markers, Graph Paper if available or Coordinate plane board, Straight edge
- Student Practice Packet

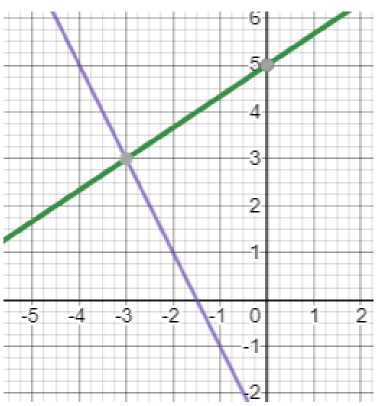
Student Materials:

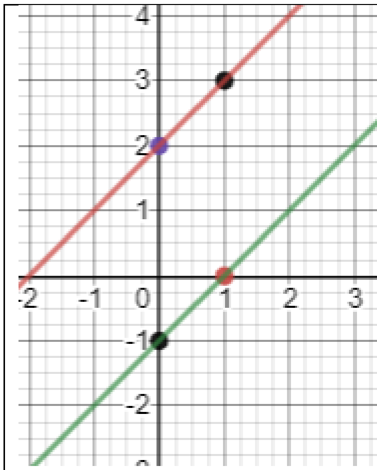
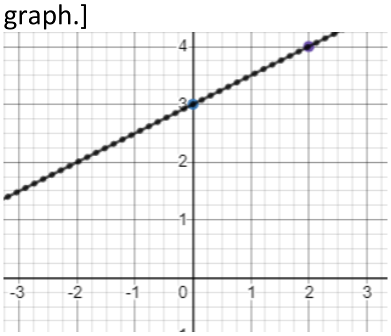
- Paper and a pencil, and a surface to write on
- Calculator not required but may be used to check calculations.
- Optional but helpful: Straight edge, Graph Paper

Note: There are a charts and graphs that will need to be prepared ahead of time to show to students during the lessons this week. In today's lesson, most of the systems will fit on a 5 x 5 coordinate plane, but a couple will need a 10 x 10.

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Week 3 of Tennessee's At Home Learning Series for math! Today's lesson is for all our 8th graders out there, though all students are welcome to tune in. This lesson is the twelfth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. If you don't already have the student packet for this lesson, you can find it online at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.**	Students get materials ready for the lesson.

Today we will be continuing our learning about systems of linear equations, and we will be working on estimating solutions by graphing! Before we get started, to participate fully in our lesson today, you will need: • **Paper and a pencil, and a surface to write on** • **A calculator is not required but may be used to check calculations.** • **Optional but helpful materials would be a straight edge and graph paper** **Ok, let's begin!**	
Intro (2 min) **We are in Week 3 of our 8th grade mathematics learning series, and we are working with equations and expressions this week. Let's warm with a little mathematical practice.** **In Lesson 11, we looked at this chart about maximum heart rates based on a person's age.** [Teacher shows the chart.] **Heart Rates (beats per minute)** **Maximum Heart Rate (MHR)** — **220 - age** **Vigorous Intensity Exercise** — **70-85% of MHR** **Moderate Intensity Exercise** — **50-70% of MHR** **Remember that *x* represents the age of a person, and *y* represents the maximum heart rate. Here's what we wrote in Lesson 11** [Write on whiteboard.] **$y = 220 – x$.** **Now, let's extend this to write and simplify an expression that gives the heart rate that is 75% of the maximum heart rate for a person who is *x* years old. Think about how we would modify the equation.** [pause] **If we want to represent 75% of the maximum heart rate, we can write the equation this way:** [Teacher writes/shows & speaks.] **$y = 0.75(200 – x)$** **For a quick practice, let's evaluate this equation for someone who is 25 years old and someone who is 55 years old.** [pause] **Check your work with mine.** Teacher writes/shows and speaks.] **$y = 0.75\ (200 – 25) = 0.75(175) = 131.25$ beats per minute for a 25 year old.**	Students listen to the problem, consider what the problem is asking, and determines possible solution strategies.

y = 0.75(200 – 55) = 0.75(145) = 108.75 beats per minute for a 55 year old. **Great warm-up! Now, let's let started with today's focus.**	
Teacher Model (10-12 min) Objective 1: Examine the graph of a system of linear equations to determine the solution where the solution to a linear system is the point or points at which the lines intersect. **As we begin, let's remember that a system of linear equations is two or more linear equations with the same variables. In Lesson 11, we inspected the equations and the graphs to determine if the systems had one solution, no solution, or infinite solutions. We're going to expand on that today.** **Let's look at three graphs. You can sketch a picture of each example to help you remember this later.** **Here's Graph A:** [Show the completed system graph.]  **What do you notice on this graph?** [pause] **These lines intersect at one point, and from Lesson 11, we know that this means the system has ONE solution. Now, let's look closer. Can we estimate the ordered pair of the location of the intersection?** [Point at the location (-3, 3).] **Yes, we can! This point looks like it is at the ordered pair (-3, 3). So, the solution to this system of linear equations, the ordered paired where that makes both questions true, is (-3, 3). You can sketch a sample of this system on your paper.** **Next, let's look at Graph B:** [Show the completed system graph.]	Objective 1: Students will examine graphs of systems of linear equations to identify the solution to the system.

 **What do you notice about this graph?** [pause] **Right! These lines are parallel. They will never intersect. Recall also from Lesson 11 that this means the system has NO solutions. There is no point of intersection that makes both equations true. Don't forget to sketch a sample of this type of system.** **Finally, let's look at Graph C:** [Show the completed system graph.]  **What do you notice here?** [pause] **I hope you noticed that there's only one line. In this scenario, the two equations in the system actually represent the same line. We learned in Lesson 11 that this means there are INFINITE solutions to this system meaning that EVERY ordered pair that represents a point on the line makes both equations true. Make sure you sketch a sample of this system on your paper as well.**	
Objective 2: Example 1: Graph systems of linear equations on a coordinate plane and examine the resulting graph to determine the solution.	Objective 2: Students will review using the y-intercept and slope of a linear equation to graph a model of the equation on a coordinate plane. Then, students will graph the system on the same coordinate plane and examine the resulting graph to determine the solution.

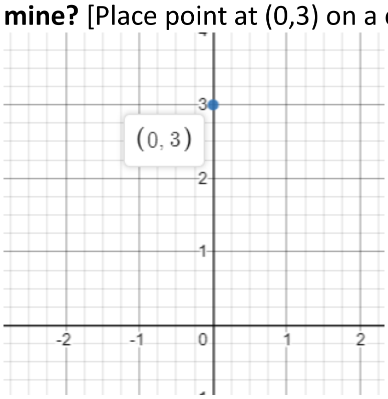
Now, let's extend this with a couple practice problems, but we will be graphing the linear equations on a coordinate plane ourselves to help find our solutions.
Let's think about this. What is the easiest form of an equation to graph from? [pause] **That's right! Usually, if we transform the equation into slope-intercept form, it becomes fairly straight forward to graph.**

Let's start by graphing this system of equations on the same coordinate plane. [Teacher writes/shows & speaks each equation.] **To get started, draw a coordinate plane on your paper and write down this system of equations.**

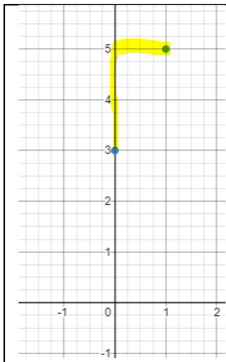
$$y = 2x + 3$$
$$y = \frac{2}{3}x - 1$$

Now, both of these equations are already in slope-intercept form. So, let's recall how to graph these on a coordinate plane.

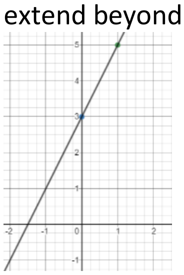
For the first equation, can you identify the y-intercept? [pause] **That's it…3!** [Point to 3 in the first equation.] **This means we can place a point on the coordinate plane at (0, 3) Can you graph that on your coordinate plane as I graph it on mine?** [Place point at (0,3) on a coordinate plane.]



Now, what is the slope of the line? [pause] **Excellent…it is 2!** [Point to the coefficient of 2 in the first equation.]
Remember that slope is also described as rise over run. The value of 2 can be written as $\frac{2}{1}$. So, we will put the next point on the graph by moving up two from the y-intercept and to the right 1. Graph the next point on your graph as I work on mine. [Using your marker, start at the y-intercept value and physically move the marker up two and to the right one and plot the next point at (1,5).]



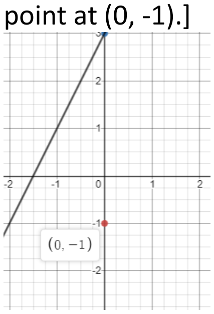
Now, using a straight edge if you have one, draw a line through these two points to the edge and extend beyond these two points to complete the graphic model of the linear equation. [Connect the two points with a straight line and extend beyond the points to indicate that the line continues.]



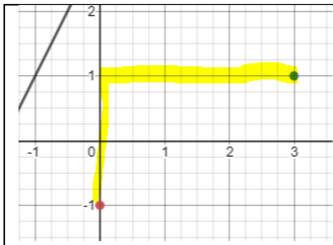
Now, let's look at the second equation of

$$\boldsymbol{y = \frac{2}{3}x - 1}$$

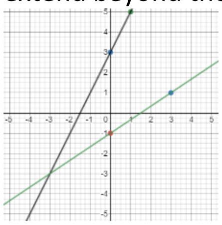
What is the y-intercept? [pause] **That's it…-1. So, let's plot the point (0,-1) on the same graph as the first equation.** [Plot point at (0, -1).]



What is the slope of the second line? [pause] **You got it! The slope of this line is $\frac{2}{3}$ so the next point is 2 units up and 3 units to the right of the y-intercept. Graph your next point as I graph mine.** [Using your marker, start at the y-intercept value and physically move the marker up two and to the right three and plot the next point at (3, 1).]



Now, using a straight edge if you have one, draw a line through these two points to the edge and extend beyond these two points to complete the graphic model of the linear equation. [Connect the two points with a straight line and extend beyond the points to indicate that the line continues.]

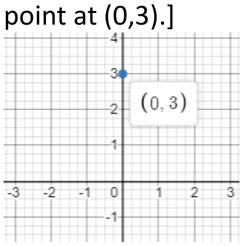


Great work! This is a complete graph of this system of equations.

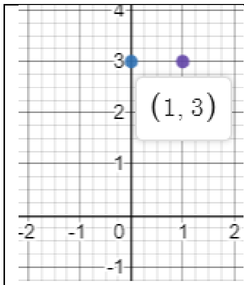
Let's look at another system to practice graphing. [Teacher writes/shows & speaks.] **Draw another coordinate plane to get ready.** [Pause] **Okay! Here is our next system. Go ahead and write these down.**

$$\boldsymbol{y = 3}$$
$$\boldsymbol{y = -\frac{1}{2}x + 2}$$

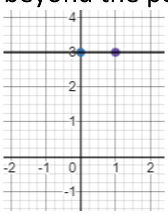
The first equation is a little different. This equation has a y-intercept of 3 [Point at 3 in the first equation.]**, but it doesn't have a slope. What does that mean? Do you remember?** [pause] **That's it! The slope is zero. This means that this equation represents what type of line?** [Pause] **Did you say a horizontal line? You are correct! First, we plot the y-intercept at (0,3).** [Using a new coordinate plane, plot the point at (0,3).]



Then, because this is a horizontal line with zero slope, we can simply put a second point to the right of the y-intercept. You make sure to draw along with me. [Plot a point at (1,3) moving your marker physically from the y-intercept to the next point.]



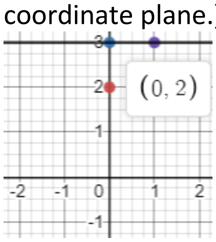
Now, connect the points across the coordinate plane to represent the line y = 3.
[Connect the two points with a straight line and extend beyond the points to indicate that the line continues.]



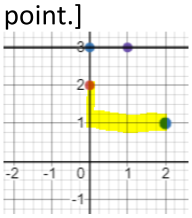
Finally, let's graph that second equation.

$$y = -\frac{1}{2}x + 2$$

What is the y-intercept? [pause] **You got it! The y-intercept is at (0,2), and the slope? Yes! The slope is $-\frac{1}{2}$.**
Let's plot the y-intercept first. [Plot point at (0,2) on the same coordinate plane.]

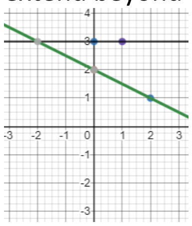


Now, let's think about the slope of $-\frac{1}{2}$. Since the slope is negative, we can start at the y-intercept and move down one (the rise is negative) and then to the right 2. If the points are plotted correctly, a line with a negative slope should slant down as you look at it from left to right. [Plot a point at (2,1) moving your marker physically from the y-intercept to the next point.]

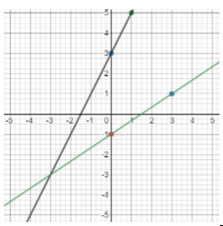


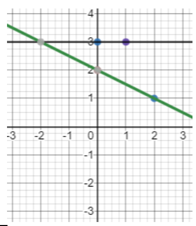
Now, using a straight edge if you have one, draw a line through these two points to the edge and extend beyond these two points to complete the graphic model of the linear

equation. [Connect the two points with a straight line and extend beyond the points to indicate that the line continues.]



Let’s look at both of the systems that we just graphed. [Teacher shows both systems.]





We can see that in both of these graphs, the lines intersect at one point. [Point to the point of intersection in both graphs.] **This tells us that each system has ONE solution, and the solution is the ordered pair of the point of intersection.**

In the first system, where do the two graphs cross? [pause] **The point of intersection is at (-3, -3).** [Point to this location on the first system graph.] **That means the solution to the system of equations is (-3, -3). Let’s write that under the graph.**
What about the second system? [Pause] I**n the second system, the point of intersection is at (-2, 3). That means the solution is (-2, 3). Let’s write that under the graph.**
[Point to this location on the second system graph.]

Now that we’ve practiced graphing linear equations, let’s look at some additional systems and look for solutions to the system. Draw another coordinate plane to get ready. [Pause] **Okay! Here is our next system.** [Write/show and speak.]

$$y = x - 4$$
$$2x - 2y = -2$$

As we look at this system, we notice that the first equation is in slope-intercept form, but the second equation is not. Before we graph the system, what should we do? [Pause] **That’s it! Let’s transform the second equation. Follow along**

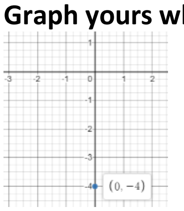
with me, but I think you will remember how to do this. [Write/show and speak.]

$$2x - 2y = -2$$
$$2x - 2x - 2y = -2 - 2x$$
$$-2y = -2 - 2x$$
$$\frac{-2y}{-2} = \frac{-2}{-2} - \frac{2x}{-2}$$
$$y = 1 + x \text{ or } y = x + 1$$

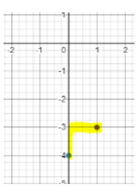
Now, we are ready to graph the system. Let's start with the first equation

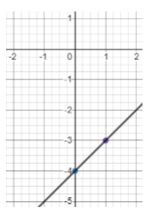
$$\boldsymbol{y = x - 4}$$

Can you identify the y-intercept? [pause] **You got it! The y-intercept is -4. So, we will plot this at (0,-4).** [Plot (0,-4).] **Graph yours while I graph mine.**



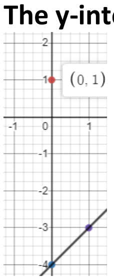
The slope of the line is one which we can express as a fraction of $\frac{1}{1}$. This means that our rise over run is up one from the y-intercept and to the right 1. Let's plot the next point and then connect the points with a line. [Plot the second point and connect the points extending the line beyond the points.]



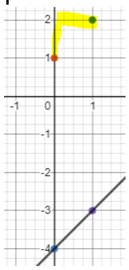
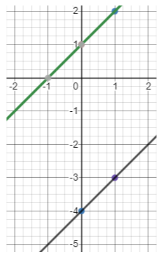


Now, let's graph the second equation on the same coordinate plane using the transformed equation $y = x + 1$.

The y-intercept is 1. So, we will plot this at (0,1). [Plot (0,1).]



The slope of the line is one which is the same as the first equation. We can plot the next point that is one unit up and one unit to the right of the y-intercept and then connect the

<table>
<tr>
<td>
points with a line. [Plot the second point and connect the points extending the line beyond the points.]
 

What do you notice about these lines? [pause]
Did you see that the equations had the same slope but different y-intercepts? In Lesson 11, we said that a system like this would have NO solutions because the lines were parallel. Therefore, the lines would never intersect. This is a graphical model of a system of linear equations that have NO solutions.

Remember these key points. You can write these down to help you remember later. [Teacher writes or shows these.]

If the equations in a system of linear equations have different slopes, they will have one solution. The solution is the ordered pair of the point of intersection in the graph.
If the equations in a system of linear equations have the same slope but different y-intercepts, they will have no solutions. The lines do not intersect on the graph.
If the equations in a system of linear equations are the same line, they will have infinite solutions. The equations are the same line.

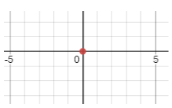
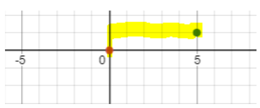
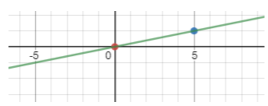
Let’s look at a few more, and you work along with me.
</td>
<td></td>
</tr>
<tr>
<td>
<u>Guided Practice </u>(10-12 min)

[I Do]
I’ll walk through one more for you – this time in context. Lynn is choosing a new music subscription plan. Company A offers a plan for no monthly fee but does charge 20 cents a song for downloads. Company B offers a $1 monthly fee but only 10 cents per download. If we let “x” be the number of downloads and “y” be the total monthly cost of the plan, then the equations of this system would look like this:
[Write/show & speak.]
</td>
<td>Students use their knowledge of transforming equations to slope intercept form to graph the system of equations to determine if each system has one, none, or infinitely many solutions.</td>
</tr>
</table>

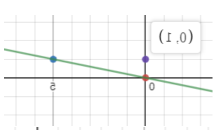
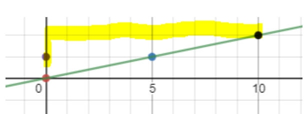
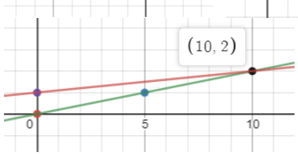
Company A: y = 0.20x
Company B: y = 0.10x + 1

Let's graph this system of equations, and see if we can determine when the subscription plans would cost the same per month. You draw a coordinate plane on your paper and follow along with me.

Let's start with Company A. The y-intercept is zero, and the slope is 0.20. How could we represent this as a fraction for a rise over run? [pause]
Did you remember that 0.20 is the same as 20/100 or 1/5? [Pause and write the equivalent statement 0.20 = 20/100 = 1/5.]
Great! Now, we can use the fraction 1/5 for our slope on the graph. So, let's plot the y-intercept and use the slope of the line to get our second point to graph. [Plot the point (0, 0) and then move up one unit and right 5 units to the next point on the line at (5,1) , then draw the line.]

Now, in the second equation for Company B, the y-intercept is 1, and the slope is 0.10. How could we represent this as a fraction for rise over run? [pause]
Yes! 0.10 is the same as 10/100 or 1/10 [Write the equivalent statement 0.10 = 10/100 = 1/10.] **So, let's plot the second line on the coordinate plane.** [Plot the point (0,1), and then move up 1 unit and to the right ten units at (10,2), then draw the line.]

If you carefully draw the lines, you should see that they intersect at one point. So, we would say that this system has ONE solution. Look carefully at the graph. Where is that point of intersection? [Pause, then point at the intersection.]
This is where the equation lines intersect. The coordinate pair that represents where they intersect is (10,2).
Now, what does that mean in context? [pause]

If you said that it would take 10 song downloads for the subscription services to cost the same, then you are right! If you downloaded 10 songs in a month, you would be charged $2 for the month in either plan.

Now, let's start working through one together.
[We Do]
Let's look at this system of equations. Make sure to have your coordinate plane drawn on your own paper and write down this system.
[Teacher writes/shows this system.]

$$\mathbf{3x + 4y = 12}$$
$$\mathbf{y = -\frac{3}{4}x + 3}$$

Are the equations ready to graph? [pause]
Right! The first equation needs to be transformed into slope-intercept form. I'll give you a minute to do that. [pause]
Did you solve it this way? [Write/show & speak.]

$$\mathbf{3x + 4y = 12}$$
$$\mathbf{3x - 3x + 4y + 12 - 3x}$$
$$\mathbf{4y = 12 - 3x}$$
$$\mathbf{\frac{4y}{4} = \frac{12}{4} - \frac{3x}{4}}$$
$$\mathbf{y = 3 - \frac{3}{4}x \text{ or } y = -\frac{3}{4}x + 3}$$

So, there is something interesting that occurs here. Take a look at the equation we just transformed and the one that we didn't need to. [pause] **Did you notice that the equations are actually the same?** [pause] **So, what does that tell us about the graph of the system and solution of the system?**[pause]
Exactly! We don't even have to graph this system. It will be the same line. Therefore, that means there are INFINITE solutions. Every ordered pair that represents a point on the line is a solution.

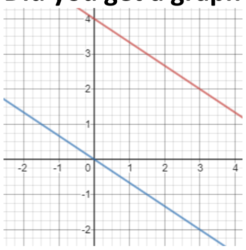
[You Do]
Terrific work! Now, here's one more for you to try mostly on your own. How many solutions does this system have? Think about how you know. Make sure you have your coordinate plane ready along with writing down the system.
[Write/show & speak.]

$$\mathbf{2x + 3y = 12}$$
$$\mathbf{y = -\frac{2}{3}x}$$

[Pause, give time for student to work through this problem and graph.]

Okay! What did you discover about these equations? [pause]

Did you get a graph that looks like this? [Show graph.]



You should have transformed the first equation into $\boldsymbol{y = -\frac{2}{3}x + 4}$ [Write equation while speaking.]

Then, since both of the equations have the same slope but different y-intercepts, these are parallel lines. On the graph, they will never intersect. Therefore, we say that this system has NO solutions. Is that what you came up with too? [pause]

Great! This should be coming together for you now. Keep in mind that the solution of a system of linear equations is the point of intersection of the lines defined by the equations.

<u>Additional Problems if needed:</u>

Let's try this system of equations. [Teacher writes/shows & speaks.]

$$\boldsymbol{y = \frac{2}{3}x + 5}$$

$$\boldsymbol{y = -2x - 3}$$

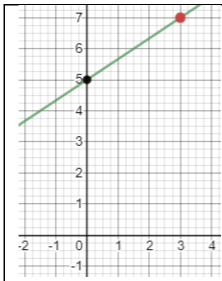
Are the equations in a form where we can inspect or look at the equations to find the slope and y-intercept? [pause]

If you said "yes", then you are right! Let's go ahead and graph this system.

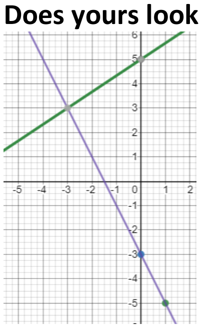
Starting with the first equation, find the y-intercept and the slope.[pause]

Right! The y-intercept is at (0,5) and the slope is 2/3 so you would move two units up from the y-intercept and three units to the right to get a second point to be able to draw the line. Take a minute to do that, and then we'll check against mine. [pause]

Did you come up with a line that looks like this? [Show graph.]



Terrific! Now, let's graph the second equation on the same coordinate plane. What is the y-intercept and slope? [pause]
If you said the y-intercept is at (0, -3) and the slope is -2 which means from the intercept you move two units down and 1 unit to the right, then you've got it! Go ahead and graph that. [pause]
Does yours look like this? [Show graph.]



How many solutions does this system have? [pause]
Absolutely, correct! It has ONE solution since the lines intersect at one point on the graph. Can you read the ordered pair of the point of intersection? [Point at the intersection on the graph and pause.]
I see that this system intersects at (-3, 3). So, the solution to this system is the ordered pair (-3, 3).

One last system to look at: [Write/show & speak.]

$$\boldsymbol{y = x - 1}$$
$$\boldsymbol{2y = 2x + 4}$$

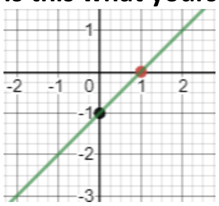
Are these equations ready to graph? Do I need to transform one or both of them into slope-intercept form? [pause]
If you said the second you, you are right! Go ahead and transform it. Take a minute to work through it, and we'll check it against mine. [pause]
OK, this is what you should have: [Write/show and speak.]

$$2y = 2x + 4$$
$$\frac{2y}{2} = \frac{2x}{2} + \frac{4}{2}$$
$$y = x + 2$$

Now, we should be ready to graph. Let's start with the first equation of y = x – 1. What is the y-intercept and slope? [pause]

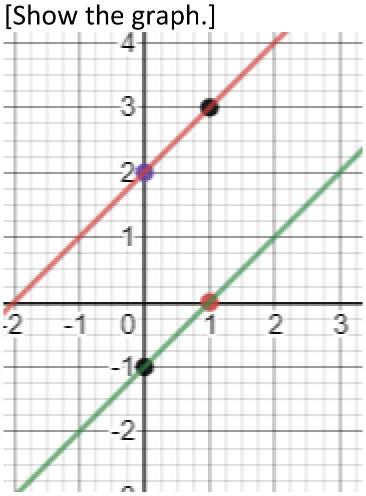
Right! The y-intercept is at (0,-1) and the slope is 1 where you would move up one unit from the intercept and to the right one unit to mark the second point to draw the line. Take a minute to do that. [pause]

Is this what yours looks like? [Show the graph.]



Great! Now, using the transformed second equation, plot the equation on the same coordinate plane. Identify the y-intercept and slope first! Give it a try! [Pause – give a little extra time.]

Is this what your system looks like? Check it against mine. [Show the graph.]



So, what does this tell us about the solution to this system? [pause]

Right! The lines are parallel and will never intersect. This tells us that there are NO solutions to system of equations.

Terrific work! I hope you found a straight edge because it is easier to get the lines to draw straight if you have one – especially since not every system intersects at integer values!

<u>Independent Practice</u> (1 min) **Superb work today, students! Today, we explored Solving Systems of Linear Equations by Graphing. After this lesson, you will have a few problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, <u>www.tn.gov/education</u>.** [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	
<u>Closing</u> (1 min) **I enjoyed graphing systems of linear equations with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	