Math: Grade 8, Lesson 14, Solving Systems by Elimination Method using Addition and Subtraction

Lesson Focus: Solving Systems by Elimination Method using Addition/Subtraction
Practice Focus: Students will use elimination to solve a system of linear equations by adding and subtracting the equations to eliminate one variable. Students can then use the resulting equation to solve for the remaining variable or determine if there is no solution, one solution or infinite solutions.
Objective:

- Understand how the process of elimination can be used to solve a system of linear equations using the addition and subtraction method.
- Apply this understanding to solve mathematical and real-world problems

Key Vocabulary: Elimination, coefficients, addition property of equality, subtraction property of equality
TN Standards: 8.EE.C.8 – solve systems of equations using elimination
Teacher Materials:

- Whiteboard and Markers
- Student Practice Packet

Student Materials:

- Paper and a pencil, and a surface to write on
- Calculator not required but may be used to check calculations.

Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Week 3 of Tennessee's At Home Learning Series for math! Today's lesson is for all our 8th graders out there, though all students are welcome to tune in. This lesson is the fourteenth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. If you don't already have the student packet for this lesson, you can find it online at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.** **Today we will be learning about systems of linear equations, and we will be working on estimating solutions by inspection! Before we get started, to participate fully in our lesson today, you will need:** • **Paper and a pencil, and a surface to write on** • **A calculator is not required but may be used to check calculations.**	Students get materials ready for the lesson.

Ok, let's begin!	
Intro (2 min) **You may recall that in earlier lessons, we used this chart about maximum heart rates based on a person's age.** [Show the chart.] **Heart Rates (beats per minute)** **Maximum Heart Rate (MHR): 220 - age** **Vigorous Intensity Exercise: 70-85% of MHR** **Moderate Intensity Exercise: 50-70% of MHR** **Remember that x represents the age of a person, and y represents the maximum heart rate. Here's what we wrote in Lessons 11 and 12:** [Write the problem on the whiteboard.] $y = 220 - x$ **In this equation, x represents the age of a person, and *y* represents the maximum heart rate.** **Let's work another problem related to this same chart.** [Teacher will write/show and speak.] **Cassandra and Jasmine like to run together. They both try to maintain a heart rate that is 60% of their maximum heart rates, meaning they are trying to keep their heart rate at a certain number while they are running. Jasmine is 5 years older than Cassandra, so she thinks her 60% of MHR must be 5 beats per minute lower than Cassandra's. Do you agree?** **To answer that, let's first decide what expressions we'll use for Cassandra and Jasmine's ages. We can use *x* to represent Cassandra's age, and *x* + 5 for Jasmine's age, because she is five years older.** [Teacher will write and speak.] **What expressions show 60% of MHR for each person?** [Teacher will speak and write.] **For Cassandra, we can use:** $0.6(220 - x)$ **For Jasmine:** $0.6(220 - (x + 5))$ **We don't know their exact ages, but remember, we're trying to figure out whether or not 60% of Jasmine's MHR is 5 beats per minute lower than Cassandra's, so the actual age of the two people does not matter, just the difference between**	Students listen to the problem, consider what the problem is asking, and determines possible solution strategies.

Heart Rates table (as printed within the cell):

Heart Rates (beats per minute)	
Maximum Heart Rate (MHR)	**220 - age**
Vigorous Intensity Exercise	**70-85% of MHR**
Moderate Intensity Exercise	**50-70% of MHR**

their ages. You can pick any two ages that are 5 years apart, and it will solve the problem. **So using the expressions for each person, we get the following:** **For Cassandra:** **0.6(220 – *x*) or 132 – 0.6*x*** **For Jasmine:** **0.6(220 – (*x* +5)) or 129 – 0.6*x*** **Then we want to solve for Jasmine's heart rate:** **(132 – 0.6*x*) – (129 – 0.6*x*) = 3** **Jasmine's heart rate is 3 beats per minute lower than Cassandra's, not five, so we would have to disagree with Jasmine's original assessment!** **That was a good warm-up. So, let's look at some other problems. Are you ready?** [Pause] **OK, let's get started!**	
Teacher Model (10-12 min) Objective 1: Example 1: Solving a system of equations by elimination using the addition/subtraction method **Eliminate means to get rid of something – simple enough, right? But in mathematics, the elimination method is a great way to solve a system of linear equations. Elimination can be used to solve a system of linear equations by adding or subtracting the equations to eliminate one variable, and using the resulting equation to find the other variable, or identify if there's no solution or an infinite number of solutions.** **As we work through the problems, keep in mind that a system of equations must have one pair of terms with coefficients that are the same or opposites for a term to be eliminated by adding or subtracting.** **Let's use elimination to solve some problems. Remember from the previous lessons, listen to the information first, then grab your pencil and paper to follow along. First up, we're going to try to solve a riddle:** [Teacher will read/show problem.] **Two times a number *x* plus a number *y* equals eight. Four times the number *x* minus the number *y* equals four.**	Objective 1: Example 1: Students will review how to write equations to represent a situation, then use the elimination method with addition and subtraction only to solve the system of equations.

How can we use this system of equations to solve the riddle? [Teacher will write/show the system of equations.] $\mathbf{2x + y = 8}$ $\mathbf{4x - y = 4}$ **OK, try to write along as we eliminate one of the variables.** [Teacher will write/show.] $\mathbf{2x + y = 8}$ $\underline{\mathbf{+\ 4x - y = 4}}$ $\mathbf{6x + 0 = 12}$ $\mathbf{6x = 12}$ $\mathbf{\frac{6x}{6} = \frac{12}{6}}$ $\mathbf{x = 2}$ **As you can see, we eliminated the *y* variable here. This helps solve the system because the result is an equation with only one variable, *x*, which can then be solved to find the value. Then, the value of x can be substituted into one of the equations to find the value of *y*. This will give us a single solution to the problem.** **You can also see that the sum of the coefficients of the *y* terms is 0. The Addition Property of Equality allows us to add equal values to both sides of the equation to eliminate *y*.** **Now that we have *x*, let's solve for *y*. Follow along with me. Remember to substitute the value of *x* into either of the equations, like this:** [Teacher will write/show.] $\mathbf{2x + y = 8}$ $\mathbf{2(2) + y = 8}$ $\mathbf{4 + y = 8}$ $\mathbf{y = 4}$	
Objective 1: Example 2: Solving using the elimination method and checking the solution by substitution **Let's look at another problem. Write down this system of equations:** [Teacher will write/show.] $\mathbf{2r + 3s = 14}$ $\mathbf{6r - 3s = 6}$ **How should we go about solving this? Well, we can use addition to eliminate *s* and solve for *r*. As I write this down, try to follow along and see if you get the same result:** [Teacher will write/show.]	Objective 1: Example 2: Students will use the elimination method to solve a system of equations and check their solutions by using substitution.

$2r + 3s = 14$ **$6r - 3s = 6$** **$8r + 0 = 20$** **$r = 2.5$** **Did you get the same answer? Now that we have *r*, we can solve for *s*. See if you can follow along:** [Teacher will write/show.] **$2r + 3s = 14$** **$2(2.5) + 3s = 14$** **$5 + 3s = 14$** **$3s = 9$** **$s = 3$** **Let's look at this one a little closer. Are there opposite terms in the system of equations?** [pause] **Yes, the opposite terms are 3*s* and -3*s*. What would be a way to check your solution?** [Pause] **You could substitute *r* = 2.5 and *s* = 3 into the second equation, to make sure it can be solved correctly.** **Now that we've done a few problems using elimination, can you tell me when you know to add the equations in a system?** [pause] **That's right, when they contain a pair of opposite terms. When would you use subtraction?** [pause] **You got it! When the system contains a pair of equivalent terms.** **Let's work a few more problems, and this time we'll focus on solving by subtracting.** Objective 1: Example 3: Does the order in which you subtract the equations matter? **Listen to the following problem, then get that pencil and paper ready! Be thinking about what information you'll need to write your system of equations as I read the problem.** [Teacher will speak and write/show.] **Hannah is helping her younger brother learn to count money. She gave her brother a total of 20 coins – all nickels and pennies. Hannah's brother said he counted 68 cents. How can Hannah solve a system of equations to tell whether he counted the money correctly?** **First, let's write our system of equations to relate the number of nickels and pennies. We'll let *x* = the number of pennies, and *y* = the number of nickels. We know that the total number of coins is 20, and the total value of the coins is 68 cents.** [Write/show the system of equations.] **$x + y = 20$**	Objective 1: Example 3: Students will discover the solution is the same because changing the position of the equations when you use elimination does not change the solution.

***x* + 5*y* = 68** **Did you get it right? Great! Now we want to eliminate a variable. The difference of the coefficients of x is 0. Let's apply the Subtraction Property of Equality to subtract the equations to eliminate x. We will subtract each term of the second equation from the like term in the first equation.** [Teacher will write/show.] $x + y = 20$ $-(x + 5y) = -68$ $0 - 4y = -48$ $y = 12$ **Now, that we have y, we can solve for the other variable. Follow along as we write out the answer:** [Teacher will write/show.] $x + y = 20$ $x + 12 = 20$ $x = 8$ **If Hannah gave her brother 8 pennies and 12 nickels, then he definitely counted correctly! Would the solution be different if you subtracted the first equation from the second equation?** [Pause] **No, it should be the same, because changing the position of the equations when you use elimination does not change the solution.** **Let's work this out to prove it:** [Teacher will write/show.] $x + 5y = 68$ $-(x + y) = -20$ $0 + 4y = 48$ $y = 12$ **Let's look at a few more, and you work along with me.**	
<u>Guided Practice</u> (10-12 min) [I Do] **I'll walk through one more for you, but if you want to get your pencil and clean paper, you can follow along. We're going to us elimination to solve the following system of equations:** [Write/show and speak the problem.] $3x + 2y = 17$ $6x - 2y = 28$	Students use their knowledge of solving systems of linear equations by elimination (using addition/subtraction strategies) to solve the systems.

First, of course, we want to eliminate a variable. We can use the Addition Property of Equality to add equal values to both sides of the equation to eliminate *y*. [Teacher will write/show and speak.]

$$\begin{array}{r} 3x + 2y = 17 \\ \underline{+\ 6x - 2y = 28} \\ 9x \qquad = 45 \\ x = 5 \end{array}$$

Now that we have our value for x, we can then substitute and solve for y. It will look like this: [Write/show and speak the problem.]

$$\begin{aligned} 3x + 2y &= 17 \\ 3(5) + 2y &= 17 \\ 15 + 2y &= 17 \\ 2y &= 2 \\ y &= 1 \end{aligned}$$

Now we have found our values: [Write/show and speak the problem.]

$x = 5$
$y = 1$
or we could write the ordered pair like this (5, 1)

[We Do]
Now, here's one more for you to try mostly on your own. If you have a calculator and want to use it to check your work, that's OK. [Write/show and speak the problem.] **Two balloons, Balloon A and Balloon B, have a total volume of $\frac{3}{5}$ gallon. Balloon A has a greater volume than Balloon B. The difference of their volumes is $\frac{1}{5}$ gallon.**

Here I've got a system of equations where *a* = the volume of Balloon A and *b* = the volume of Balloon B. Can you fill in the blanks? Write this on your paper and try to finish it. [The teacher will write/show and then pause to let the student think of the missing parts.]

$a + b =$ ___
$a - b =$ ___

[Pause] **Did you get something like this?** [Write/show and speak the following.]

$a + b = \frac{3}{5}$

$a - b = \frac{1}{5}$

Great! We want to use elimination to solve this system of equations. Can you add the equations to eliminate *b* and find the value of *a*? [Write/show with blanks.] $a + b = \frac{3}{5}$ $a - b = \frac{1}{5}$ $\_\_\ a + \_\_\ b = \_\_\_$ $a = \_\_\_$ [Pause] **Did you get something like this?** $a + b = \frac{3}{5}$ $a - b = \frac{1}{5}$ $2a + 0b = \frac{4}{5}$ $a = \frac{2}{5}$ **Now that we can substitute $\frac{2}{5}$ for *a*, how can you find the value of *b*? What is the value of *b*?** [Pause] **See if you can figure out by filling in the blank:** [Teacher will write/show.] $\frac{2}{5} + b = \frac{3}{5}$ $b = \frac{3}{5} - \frac{2}{5}$ $b = \_\_\_\_$ [Pause, giving time for student to answer.] **So what's the final answer?** [Pause] **If you said the volume of Balloon A is $\frac{2}{5}$ gallon and the volume of Balloon B is $\frac{1}{5}$ gallon, then you are correct! You're doing great at this!** [You Do] **Here's one for you to try mostly on your own, so you're your paper and pencil ready, and listen carefully:** [Teacher will read/show the problem.] **At a basketball game, a team made 56 successful shots. They were a combination of 1- and 2-point shots. The team scored**	

94 points in all. Can you use elimination to solve the system of equations and find the number of each type of shot? **How would you begin to solve this one?** [Pause] **Let's say x = the number of 1-point shots, and y = the number of 2-point shots. So, what equations can be written to represent this situation?** [Pause for student to answer.] **Did you set up equations that look like this?** [Teacher will write/show.] $x + 2y = 94$ $x + y = 56$ **Great! Now use elimination to determine how many 2-point shots were made.** [Pause] **Let's take a quick look at your progress. Did you:** [Teacher will speak and show/write.] **Begin with x + 2y = 94** **Then use subtraction like this?** $x + 2y = 94$ $\underline{-x - y = -56}$ $0 + y = 38$ **From this, you now know that there were 38 two-point shots. Now you can determine the number of 1-point shots. You can use either equation, but let's try the simplest:** $x + y = 56$ $x + 38 = 56$ **What answer did you get?** [Pause] **If you got x = 18, then you're correct!** **You really are getting this! Keep going with some independent practice.** Additional Problems (if needed): **Here's another example. Follow along on your own paper. We're going to us elimination to solve the following system of equations:** [Write/show and speak the problem.] $-6x + 5y = 1$ $6x + 4y = -10$ **First, of course, we want to choose which variable to eliminate. Looking at this system, which variable would you choose?** [Pause] **Are either variables equal or opposites?** [Pause]	

I think we can use the Addition Property of Equality to add equal values to both sides of the equation to eliminate *x*. [Teacher will write/show and speak.] **-6x + 5y = 1** **<u>+ 6x + 4y = -10</u>** **9y = -9** **y = -1** **Now that we have our value for *y*, we can then substitute and solve for *x*. Remember you can use either equation to substitute back into. Give that a try.** [Pause] **This is what I found when I used the first equation to substitute back into:** [Write/show and speak the problem.] **-6x + 5(-1) = 1** **-6x - 5 = 1** **-6x – 5 + 5 = 1 + 5** **-6x = 6** **X = -1** **Now we have found our values:** [Write/show and speak the problem.] **x = -1** **y = -1** **or we could write the ordered pair as (-1, -1).** **Here's one more example to work together. Write this system along with me.** [Teacher will write/show and speak.] **7x + 2y = 24** **8x + 2y = 30** **Which variable do we want to eliminate?** [Pause] **Did you say we could eliminate *y*?** [Pause] **That's right, but will we add or subtract in this case?** [Pause] **Right! We need to subtract because the coefficients of *y* are the same but not opposites. Let's model it this way. You write it along with me.** [Teacher will write/show and speak.] **7x + 2y = 24** **<u>-(8x +2y = 30)</u>** **-1x = -6** **x = 6** **Now that we have our value for *x*, let's substitute that back into one of the equations to find the value of *y*. You try that, and check against my work.** [Pause]	

I used the second equation this time, but remember that you can use either one. This is what my work looks like. [Teacher will write/show equations and speak.] **8x + 2y = 30** **8(6) + 2y = 30** **48 + 2y = 30 (subtract 48 from both sides)** **2y = -18 (divide both sides by 2)** **Y = -9** **We've found the solution!** [Write/show and speak the problem.] **x = 6** **y = -9** **or we could write the ordered pair as (6, -9).** **How are you feeling about your skill level now?** [Pause]	
<u>Independent Practice</u> (1 min) **Superb work today, students! Today, we explored Solving Systems of Linear Equations using the Elimination Method with addition and subtraction. After this lesson, you will have a few problems to practice on your own. I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, <u>www.tn.gov/education</u>.** [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	
<u>Closing</u> (1 min) **I enjoyed solving systems of linear equations using the elimination method with addition and subtraction with you! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	