Module 3:

Teaching Addition and Subtraction

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Objectives

- Gain an understanding of the CRA model
- Learn to use the Think Aloud strategy for math
- Learn math instructional strategies and activities to teach addition and subtraction

Standards

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Pre-K Standards	Kindergarten Standards
PK.OA.1. Represent real-world addition (putting together), and subtraction (taking from) problems up through five with concrete objects or by acting out situations.	**K.OA.1.** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
PK.OA.2. Solve addition and subtraction problems using objects for problems up through five.	**K.OA.2.** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
PK.OA.3. Compose and decompose numbers to five by using objects or drawings (may be an extension activity after reading a book).	**K.OA.3.** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing

TEAM Alignment

- Teacher Content Knowledge
- Activities and Materials
- Questioning
- Thinking

Reflection

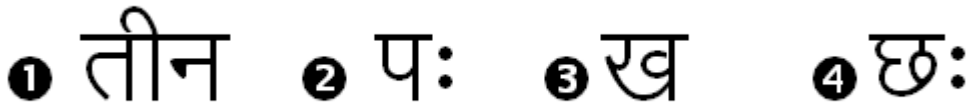
How do you teach addition and subtraction to your class? What do you do first? What steps do you take to ensure all students learn the concept? List your thoughts below:

Group Sharing

Share with your table the steps you would take to teach the lesson. Are your ideas similar to the other participants?

The CRA Method of Instruction

Look at the symbols below. Circle the two symbols that are numbers.



How did you do? Did you see any distinguishing characteristics that helped you make your determination?

The symbols look similar. What does this have to do with math? Until a child knows that five means 5 fingers, or 5 dots, or 5 toy cars, or 5 years old, the symbol doesn't look much different than a letter. It has some curved lines and some straight lines, like the letters *a, b,* or *p.*

Twenty years ago, Singapore occupied the space squarely at the bottom of the world rankings in math. Now, they are among the top. What changed? They adopted a teaching philosophy that is built on the concrete, representational, abstract (CRA) sequence of instruction. They call it CPA, with the P standing for pictorial. Regardless of the letters used, this sequence of instruction is based on the research of Jerome Bruner. It says that students must experience and interact with a concept to develop a true understanding.

Retrieved from http://www.mathcoachscorner.com/2015/05/why-cra/

CRA Video

https://www.youtube.com/watch?v=weCPBlJVSrI

Concrete-Representational-Abstract Instructional Approach

What is the Concrete-Representational-Abstract (CRA) Instructional Approach?
The CRA Instructional Approach is "an intervention for mathematics instruction that research suggests can enhance the mathematics performance of students" (Hauser). The approach is a "three-part instructional strategy, with each part building on the previous instruction to promote student learning and retention and to address conceptual knowledge" (Hauser). The three parts are as follows:

- **Concrete:** In this stage, the teacher begins instruction by modeling each mathematical concept with concrete materials. In other words, this stage is the "doing" stage, using concrete objects to model problems.
- **Representational:** In this stage, the teacher transforms the concrete model into a representational (semi-concrete) level, which may involve drawing pictures; using circles, dots, and tallies; or using stamps to imprint pictures for counting. In other words, this is the "seeing" stage, using representations of the objects to model problems.
- **Abstract:** In this stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols to represent the number of circles or groups of circles. The teacher uses operation symbols (+, –, x, /) to indicate addition, multiplication, or division. This is the "symbolic" stage, where students are able to use abstract symbols to model problems (Hauser).

In the classroom, this approach is a facilitating framework for students to create meaningful connections between concrete, representational, and abstract levels of thinking and understanding. Students' learning starts out with visual, tangible, and kinesthetic experiences to establish basic understanding, and then students are able to extend their knowledge through pictorial representations (drawings, diagrams, or sketches) and then finally are able to move to the abstract level of thinking, where students are exclusively using mathematical symbols to represent and model problems (Hauser).

Studies have shown that "students who use concrete materials develop more precise and more comprehensive mental representations, often show more motivation and on-task behavior, understand mathematical ideas, and better apply these ideas to life situations" (Hauser).

What is the Purpose of the CRA Approach?

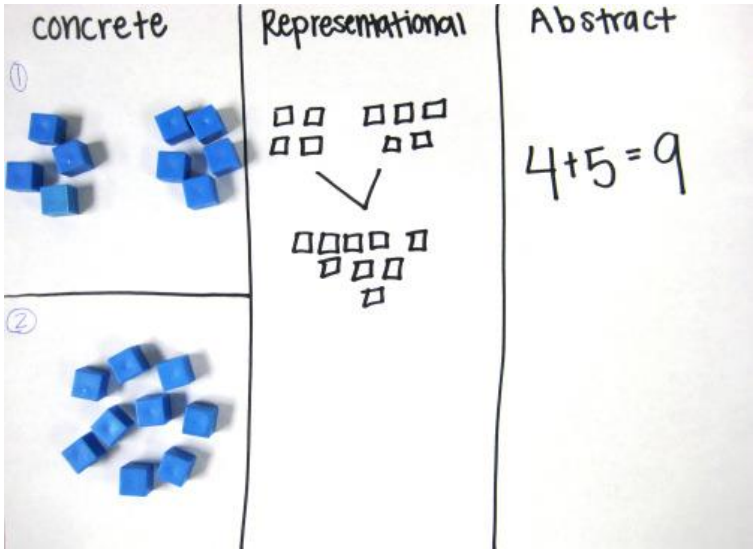
The overarching purpose of the CRA instructional approach is to "ensure students develop a tangible understanding of the math concepts/skills they learn" (Special Connections, 2005). Using their concrete level of understanding of mathematics concepts and skills, students are able to later use this foundation and add/link their conceptual understanding to abstract problems and learning. Having students go through these three steps provides students with a deeper understanding of mathematical concepts and ideas and provides an excellent foundational strategy for problem solving in other areas in the future (Special Connections, 2005).

How Can I Implement the CRA Approach in My Classroom?

One of the first and most important steps to implementing the CRA approach in the classroom is to "use appropriate concrete objects to teach particular math concepts/skills. Discrete materials (e.g. counting objects such as beans, chips, unifix cubes, popsicle sticks, etc.) are especially helpful since students can see and feel the attributes of the objects they are using" (Special Connections, 2005).

Once students have mastered the concrete level of performance, introduce appropriate drawing procedures where students problem solve through drawing simple representations of the concrete objects they previously used (e.g. tallies, dots, and circles). "By replicating the movements students previously used with concrete materials, drawing simple representations of those objects supports students' evolving abstract understanding of the concept/skill" (Special Connections, 2005).

Finally, after a student demonstrates a thorough understanding of the representative level, use appropriate strategies to help students move from that representational level to the more abstract level. If students have trouble moving to the abstract, "re-teach the mathematics concept/skill using appropriate concrete materials and then explicitly show the relationship between the concrete materials and the abstract representation of the materials" (Special Connections, 2005). If students already have a concrete level of understanding of that concept/skill, "provide students opportunities to use their language to describe their solutions and their understandings of the mathematics concept/skill they are learning" (Special Connections, 2005).



Retrieved from https://mathteachingstrategies.wordpress.com/2008/11/24/concrete-and-abstract-representations-using-mathematical-tools/

Key Idea #5

By linking learning experiences from concrete to representational to abstract levels of understanding, the teacher provides a graduated framework for students to make meaningful connections.

- Retrieved from http://fcit.usf.edu/mathvids/strategies/cra.html

Group Activity:

With the people at your table, plan a lesson using addition or subtraction based on the CRA approach. Use ideas from the lists below, or come up with your own. List the steps you would take in teaching the lesson on the chart paper provided.

Concrete Manipulative Examples:

- colored chips
- beans
- unifix cubes
- candy (ex. Skittles)
- popsicle sticks
- bears

Representational Examples:

- tally marks
- dots
- circles
- pictures of objects

Abstract Examples:

- numbers
- equations
- worksheets
- flash cards
- word problems

- Retrieved from https://makingeducationfun.wordpress.com/2012/04/29/concrete-representational-abstract-cra/

Using Think Alouds to Teach Addition and Subtraction

Jigsaw Activity

Participants will number off one through five. The same number groups will move together and read their section of the article below. After discussing the main points, participants will move back to their original table and "teach" their part of the article.

Group 1: Talk moves that engage students in discourse
Group 2: The art of questioning
Group 3: Using student thinking to propel discussions
Group 4: Setting up a supportive environment
Group 5: Orchestrating the discourse

How to Get Students Talking!
Generating Math Talk That Supports Math Learning
De Garcia, Lisa Ann.

Due to the attention in the last few years on discourse and its importance to student learning, educators nationwide are finding that they can help children become confident problem solvers by focusing on getting them to talk and communicate in partnerships, small groups, whole groups, and in writing. In addition, English Language Learners are flourishing as they experience focused opportunities for talking and trying on new mathematical vocabulary.

So what exactly is discourse? What are the teaching practices associated with successfully establishing an environment to support it, and as a result, to improve mathematical proficiency? How does one begin to elicit meaningful talk during math lessons? As a profession, we share a vision about the role student discourse has in the development of students' mathematical understanding, but are often slow to bring the students along. Children do not naturally engage in this level of talk.

This article addresses the above questions and concerns—and more. It opens with a look at discourse through NCTM's definition and its involvement with the Common Core State Standards. It then focuses on literature available on discourse, specifically the book *Classroom Discussions,* and addresses five teaching practices focused on the *how to* of getting students talking about mathematics. The article concludes with journaling insights on discourse from a

kindergarten and second-grade classroom. This article is by no means an exhaustive list of discourse "to dos;" hopefully it will however get us all started in thinking about and implementing best talk practices.

What is Discourse in the Mathematics Classroom?

NCTM's Definition

The National Council of Teachers of Mathematics (NCTM) in their 1991 professional standards describes discourse as ways of representing, thinking, talking, agreeing, and disagreeing; the way ideas are exchanged and what the ideas entail; and as being shaped by the tasks in which students engage as well as by the nature of the learning environment.

Learning from Literature on Discourse

One of the leading resources for discourse is *Classroom Discussions: Using Math Talk to Help Students Learn* (Chapin, O'Connor, and Anderson 2009). This resource and others highlight five teaching practices associated with improving the quality of discourse in the classroom.

Practice 1: Talk Moves That Engage Students in Discourse

For the first practice, the authors of *Classroom Discussions* propose five productive talk moves that can get talk going in an otherwise silent classroom:

- revoicing (by the teacher)
- restating someone else's reasoning (by the student)
- applying their own reasoning to someone else's (by the student)
- prompting for further participation (by the teacher)
- wait time (by the teacher)

The first is ***revoicing***. An example would be, "So you are saying that…" This ***revoicing*** allows the teacher to check in with a student about whether what the student said was correctly heard and interpreted by the teacher or another student. A way to encourage students to listen to their peers is through asking them to ***restate someone else's reasoning***, such as, "Can you repeat what he just said in your own words?" Another way is to ask students to ***apply their own reasoning to someone else's*** using questions such as "What do you think about that?" and "Do you agree or disagree? Why?" This helps prevent students from just thinking about what they want to share and focuses their attention on what their classmates are saying. It also helps to strengthen the connections between ideas.

Simple questions such as, "Would someone like to add on?" are ways teachers can ***prompt for further participation***. This helps elicit more discussion when not many students are talking, especially when they are not accustomed to explaining their thinking. Again it helps students to

tune in to what others are saying so that they are able to expand on someone else's idea. Perhaps the most valuable talk move suggested by Chapin, O'Connor, and Anderson is the use of ***wait time***. Often teachers are too quick to answer their own questions when no one chimes in. Children quickly become accustomed to this. Waiting provides think time and sets the expectation that someone will indeed respond and that the teacher will wait until someone does. Another important use for wait time is to provide English Language Learners, or anyone who needs extra time, with an opportunity to process the question and formulate an answer. One teacher reported that in his initial uses of wait time, one of his English Language Learners was able to participate in class discussion for the first time.

Practice 2: The Art of Questioning

Questioning is another critical component in supporting students to engage in meaningful discussions. The NCTM Standards outline roles questions have in the math classroom. The first role, ***helping students to work together to make sense of mathematics***, is addressed by the five talk moves discussed above. The second role, ***helping students to rely more on themselves to determine whether something is mathematically correct,*** can be supported by questions such as:

- How did you reach that conclusion?
- Does that make sense?
- Can you make a model and show that?
- Does that always work?
- Is that true for all cases?

Questions designed to ***help students to learn to reason mathematically*** would include:

- Can you think of a counter example?
- How could you prove that?

To ***help students to learn to conjecture, invent, and solve problems***, the teacher might ask:

- What would happen if?
- Do you see a pattern?
- Can you predict the next one?
- What about the last one?

Finally, teachers use questions to ***help students connect mathematics, its ideas and applications*** by asking:

- How does this relate to…
- What ideas that we have learned were useful in solving this problem?

Practice 3: Using Student Thinking to Propel Discussions

Because discussions help students to summarize and synthesize the mathematics they are learning, the use of student thinking is a critical element of mathematical discourse. When teachers help students build on their thinking through talk, misconceptions are made clearer to both teacher and student, and at the same time, conceptual and procedural knowledge deepens. When doing so, the teacher must be an active listener so she can make decisions that will facilitate that talk. She also needs to respond neutrally to errors, so that the students can figure out misconceptions themselves.

For example, the teacher can ask the whole class, "What do you think about that?" when a student offers an incorrect strategy or can ask the rest of the class to prove whether or not the strategy works. Through the conversation, the misconception becomes apparent to the class. This practice results in an authentic discussion focused on the mathematics and not on the individual student. The teacher also needs to be strategic about who shares during the discussion, since it is not a show-and-tell session, and choose ideas, strategies, and representations in a purposeful way that enhances the quality of the discussion.

Practice 4: Setting Up a Supportive Environment

When setting up a discourse-rich environment and one that enhances student engagement, both the physical and emotional environment must be considered. Teachers who have studied engagement find that it is very effective if students face each other, either sitting in a circle or semi-circle on the floor or sitting in chairs arranged in a circle. Teachers can sit with students as part of the circle to encourage peer-to-peer discussion.

If teachers are still having difficulty getting children to talk, they can remove themselves from the group and stand outside the circle. As a result, students are left looking only at each other, which encourages them to direct their comments to one another.

Careful consideration of the placement of visual aids and mathematically related vocabulary is important in supporting the level of talk. If charts are not visually accessible when they need to be, they will likely not be resourced by the students during whole group conversations. To increase the extent to which English Language Learners participate in group discussions, having related vocabulary and sentence frames where they can be easily accessed is critical.

For rich discussions, the emotional environment of the classroom must be safe and must be one where students want to learn and think deeply about the mathematics. When these elements are not present, the discussion stays at the surface level. Imagine a third grade classroom where the teacher introduces division for the first time and is met with cheers. It can happen! It happens when the value is on learning, challenging each other, and working together

to solve problems as opposed to just getting the right answer. For more on setting up a supportive classroom environment for discourse, see Chapter 8 of *Classroom Discussions.*

Practice 5: Orchestrating the Discourse

The teacher becomes not unlike a conductor as he supports students to deepen their understanding of mathematics through a carefully orchestrated environment. In *Orchestrating Discussions,* Smith, Hughes, Engle, and Stein outline the *Five Practices Model*, which gives teachers influence over what is likely to happen in a discussion.

The Five Practices Model

The teacher's role is to:

- Anticipate student responses to challenging mathematical tasks.
- Monitor students' work on and engagement with the tasks.
- Select particular students to present their mathematical work.
- Sequence the student responses that will be displayed in specific order.
- Connect different students' responses and connect the responses to key mathematical ideas.

Even if the teacher is focused, students still need to be held accountable. Otherwise, the discussion will be unproductive. A lot of explicit teaching must go into how to engage in each level of discussion: whole group, small group, and partnerships. In the younger grades, one will find teachers showing students exactly what they should look like and sound like when discussing their thinking. Teachers may say things like, "Today in math, we are going to practice turning and talking with our partner. When I say go, you are going to turn like this and look at your partner. When I say stop, you are going to turn around and face me. Let's practice that right now."

Even older students need to be explicitly taught what to do and say. A teacher might teach how a partnership functions by saying, "It sounds like you have an idea and you have an idea, but what seems to be lacking is for you two to put your ideas together to come up with a solution. So, what is your plan?" One very effective method of holding students accountable is to let them know exactly what they should be saying when they are talking in their partnerships or small groups. For example, "Today, when you are talking to your partners and describing your solid shapes, I expect to hear you using the words faces, edges, and vertices." It is also supportive to let students know what they should be focusing on when someone is sharing a strategy, so they have a lens for listening, which heightens the level of engagement. A teacher might say, "When

he is sharing his thinking, I want you to be thinking of how his way is similar or different to your way." Students need to be aware of themselves as learners, and a great way to heighten this awareness is through self-evaluation and goal setting. Sometimes the child is the last one to know that he is distracting or not listening. Part of developing a safe culture is supporting students in being open with each other regarding their strengths and weaknesses so they can improve their communication skills and behaviors.

It is wonderful to hear one child compliment another when she has participated for the first time or give gentle correction when another has been dominating the conversation. This level of self-awareness happens through consistent venues such as class meetings and tracking the progress of personal goals related to participation in mathematical discussions. The more students open up about themselves as learners, the deeper the relationships and, as a result, the deeper the trust.

Five Teaching Practices for Improving the Quality of Discourse in Mathematics Classrooms

1. Talk moves that engage students in discourse
2. The art of questioning
3. Using student thinking to propel discussions
4. Setting up a supportive environment
5. Orchestrating the discourse

- Retrieved from http://www.mathsolutions.com/documents/how_to_get_students_talking.pdf

After sharing your portion of the article with the other participants at your table, talk about how these strategies would work in your classroom. What listening and speaking skills would you incorporate into the Think Aloud?

Practicing Discourse with Think Alouds

The "think-aloud" process is usually introduced in four steps, gradually transferring responsibility to students:

1. The teacher reads a problem and stops as needed to explain her thoughts. Students listen. They all solve the problem together.
2. The teacher reads the problem and stops often. Students express their thoughts at each point (and often write them). The whole class, led by the teacher, solves the problem together.
3. The teacher reads the problem, allowing students to signal stopping points as thoughts occur to them. Students solve the problem individually, and then discuss their interpretations of it and solution strategies.
4. Students do this together, in pairs. They work together to solve the problem.

A benefit of this strategy for students who need additional support is that they also listen to the thinking of their classmates, enabling them to learn additional strategies from each other.

- Retrieved from http://math-problem-solving.wiki.inghamisd.org/Think-Aloud

Think Aloud video example

http://www.showme.com/sh/?h=1U4ux4S

Key Idea #6

"Thinking aloud" requires students to talk through the details of the problem, the decisions they have made as they try to solve the problem, and the reasoning behind those decisions.

- Retrieved from http://www.ldonline.org/article/63842/

Practice a “Think Aloud”

At your table, choose someone to be the teacher and practice a think aloud. Decide together on a problem that you would use with your students. How would this look different in pre-K and K? Would you use manipulatives in your Think Aloud? Why or why not?

Addition and Subtraction Games and Activities

Carousel Activity

Number off one through eight. Each number group will stand by their number chart to begin the activity. The questions are the same for addition and subtraction, with four charts for each operation. On the chart, list any ideas that you have about teaching addition and subtraction.

Chart Headings:

- What games and activities do you use to develop student's conceptual understanding?
- What concrete, representational, or reasoning strategies do you teach your students?
- What books do you use to teach addition or subtraction?
- What online resources do you use?

Common Addition Strategies

- Draw a picture
- Counting on
- Using doubles
- Fact Families (flip flop the equation)

Addition Activities in the Classroom

Guess My Number

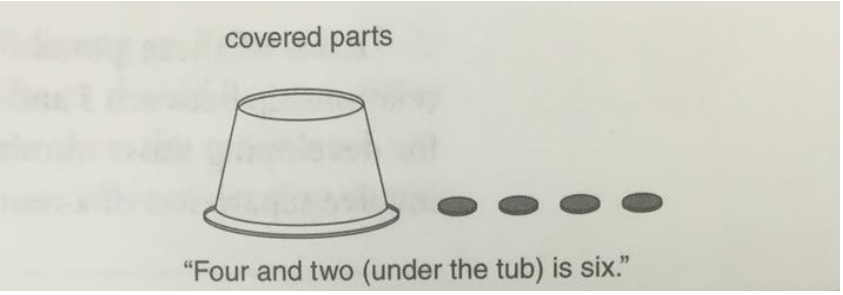
3	8
6	10

- ✓ I can use the doubles strategy to get my number.
- ✓ My number is less than seven.
- ✓ My number is 3 + 3.

1. Draw a table with numbers like the one above.
2. Give students clues, one by one so they can guess your number.
3. Use this for addition or subtraction.

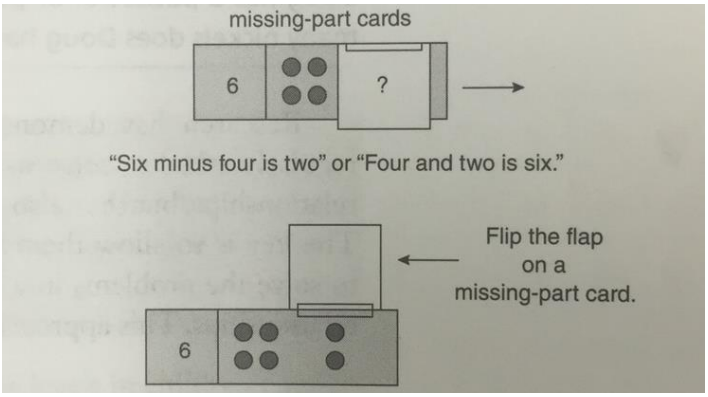
Covered Part

1. Choose a target amount.
2. A set of counters equal to the target amount is counted out, and the rest are put aside.
3. One child places the counters under a solid cup and then pulls some out into view. (This amount can be none, all or any amount in between.)
4. The partner says the two parts that makes the whole. For example, "Four and two is six"
5. If there is a hesitation or if the hidden part is unknown, the hidden part is immediately shown.



Missing Part Cards

1. For each number from four to 10, make missing part cards using strips of cardstock measuring 3 X 9 inches (this will ensure that a post it will fit over it.)
2. Each card has a numeral for the whole and two dot sets with one set covered by a post it.
3. Children use the cards by saying the two parts that make a whole such as "four and two is six."



Make and Take—Missing Part Cards

- Take five sentence strips and cut them in half.
- Use the markers and stickers to make missing part cards.
- Choose numbers that are appropriate for your classroom.

Literature Connection for Addition

Quack and Count by Keith Baker
There are seven ducklings on a page. The book shows all the ways to add numbers to make seven with a rhyming text. This book would be great to use for a **number talk** about addition. Cover the text and show the picture. Have students tell how many ducklings they see and how they see them.

Animals on Board by Stuart J. Murphy
This is a Math Start book. The illustrations show big trucks carrying animals. One truck has some animals, and then another truck comes along with some more of the same animals. The book shows the equation, but you could cover it up and ask kids how many animals are all together. You could read the book without showing the pictures at first and use the **think aloud strategy**.

Common Subtraction Strategies

- Counting Back
- Related Facts (fact families)
- The Zero Rule

Subtraction Activities in the Classroom

Songs and Finger Plays that Teach Subtraction:

- Five Little Monkeys
- Five Little Ducks
- Five Little Speckled Frogs
- There Were Ten in the Bed

Games that Teach Subtraction

Bowling

1. Set up 10 cups in a bowling pattern.
2. The first player rolls the ball and counts how many pins were knocked over and records that number on their recording sheet.
3. Using a subtraction strategy, they find out how many pins are left.
4. The student can check their answer by counting how many pins were not knocked down.
5. Now it's player two's turn!

Musical Chairs

After reading the book *Monster Musical Chairs*, set up chairs and play!
Let one group of students play the game, while the other students record the equations, and then switch.

Literature Connection for Subtraction

Monster Musical Chairs by Stuart J. Murphy
This is another Math Start book. Ten monsters are playing musical chairs and each time the music stops, one goes away. It is a great way to introduce the concept of subtraction by one.

Five Little Monkeys Sitting in the Tree by Eileen Christelow
This book is a favorite rhyming book to use with subtraction. It is fun to act out the story with the students being the monkeys.

Vocabulary Connection for Addition and Subtraction

- **Add**: putting two or more groups together
- **Subtract**: taking some away
- **Plus**: a sign that means "to add groups together"
- **Minus**: a sign that means "to take away"
- **Equals**: the total after you add or subtract
- **Sum**: how many you have in all
- **Difference**: how many you have left
- **Equation**: the number sentence

What other math vocabulary do students need to know for addition and subtraction?

Listening and Speaking Connection

How can you incorporate listening and speaking skills into the activities shared today? What questions can you ask students? What type of responses should you hear?

Possible teacher questions	Possible student responses
Can you solve this equation? How did you get the answer?	”3 + 2 = 5” I had three fingers on one hand, and two fingers on the other hand. I counted them all together.
What equations can you think of that equal 10?	Students respond by giving equations and sharing their thinking, either with addition or subtraction
I have four cookies, and I want two more. How many will I have all together?	Students respond by counting out four cubes, then adding two more, talking through their thinking as they solve the problem.

What other questions could you ask?

Writing Connection: Math Journals

- Print story problems on labels and place in student math journals. Students draw pictures and write equations to show the answer.
- Have students make up their own story problems and write them in their journals.
- Give students a prompt: How many pets do you have all together?
    - Let students work in pairs. One student writes a sentence to begin an addition story, and the partner writes the next sentence. They solve the problem together. For example: Student 1 writes: I have three cats. Student 2 writes: I have two dogs. Both students write: We have five pets.
    - Instead of writing sentences, students can also draw pictures.

Talk with the people at your table. What is one idea from this morning that you want to use in your classroom? Why?

Closing Activity

What "stuck" with you today? Write your answer on a sticky note and place it on the chart as you leave for lunch.