Tennessee Higher Education Commission

Supplementary Task: The Importance of Horizontal Asymptotes

This task is designed to support students in learning about the importance of studying horizontal asymptotes by providing a context in which the horizontal asymptote provides needed information. There are two examples in this task. One focuses on the horizontal asymptote for a rational function and one focuses on the horizontal asymptotes for a logistic function. Since the two examples are independent of each other, instructors may choose to use one or both examples as appropriate within their own existing coursework. Instructors are encouraged to adapt the examples to fit their style of teaching and their content expectations.

Example 2 includes both a table of values and a logistic model based on those values (values in the model have been rounded). An instructor may choose to provide both the table of values and the model, just the model, or just the table of values when presenting the example to students.

These examples support the objective:

> Use functions to model and solve problems using tables, graphs, and algebraic properties. Interpret constants, coefficients, and bases in the context of the problem.

in the Core to College Master Syllabus for Core-Aligned College Algebra.

Example 1: Rational Functions

A drug is injected into a patient and the concentration of the drug in the bloodstream is monitored. The drug's concentration, C(t), in milligrams per liter, after t hours is modeled by:

$$C(t) = \frac{11t}{2t^2 + 1}$$

a) Graph the function. Explain any restrictions on the domain within the context of the problem.
b) Use the graph to determine what happens as time passes and justify your response.
c) Researchers are using a new drug to treat the same condition as the drug described above. The concentration N(t) of the new drug in the bloodstream after t hours is given by:

$$N(t) = \frac{t}{3t^2 - 24t + 49}$$

where N(t) is measured in milligrams per liter. Graph this function and compare the graph of N(t) to the graph of C(t). Which drug do you think would be more effective? Explain why you chose as you did. Use the graphs to justify your response.

Example 2: Logistic Function (Exponential Functions)

The decennial population of Shelby County, according to the U.S. Census Bureau, is recorded in the table below.

Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010
Population	153,557	191,439	223,216	306,482	358,250	482,393	627,019	722,014	777,113	826,330	897,472	927,640

The population follows a logistic model, given below, where P(t) represents the population t years after 1900.

$$P(t) = \frac{1030437}{1 + \left(7.43\, e^{(-0.039\, t)}\right)}$$

a) Graph the function.
b) If the population trend described by the function continues, what will happen to the population of Shelby County as time passes? Use the graph to justify your answer.

Prior Knowledge Needed:

For example 1, students will need a basic familiarity with rational functions, including finding horizontal asymptotes.

For example 2, students will need a basic familiarity with properties of exponents and exponential functions. The fact that e has a negative exponent in the denominator may cause difficulty for some students.

Prerequisite Common Core State Standards for Mathematical Content that support these examples

Understand the concept of a function and use function notation

Interpret functions that arise in applications in terms of the context

Analyze functions using different representations

Interpret expressions for functions in terms of the situation they model

*The complete Common Core State Standards for High School Mathematics can be found in http://www.corestandards.org/Math/.

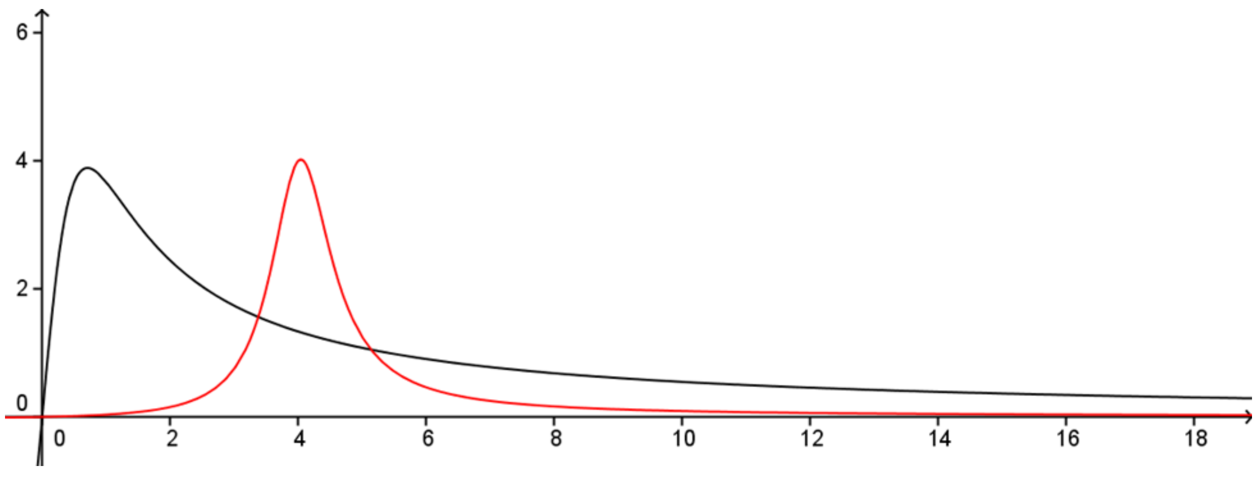
Solutions

Problem 1:

Students may approach this problem in several ways. First, they may substitute larger and larger values of t to discover what happens to C(t) as t increases. Second, they may use a graphing calculator to graph the function and determine the end behavior of the graph. Finally, they may use a theorem and compare the degree of the numerator to the degree of the denominator to determine the presence of a horizontal asymptote. In any case, students should realize that as t increases, the concentration of the drug in the bloodstream decreases and approaches 0, and students should be able to justify their conclusion using the graph of the function. (This is to be expected, as the drug will eventually "wear off.")

The graph below shows the graph of the first drug (in black) and the "new" drug (in red). As can be seen from the graph, the concentration of the first drug peaks more quickly than the concentration of the second drug. The effects from the second drug do not last as long as the effects of the first drug.

The "effectiveness" of the drug is subjective. If, for example, these drugs are painkillers, the first drug would be more effective—the pain would be lessened earlier and the effects of the painkiller would last longer. Suppose, however, that the drugs were chemotherapy drugs that have drastic side effects as long as the drug remains in the patient's system. In this case, a patient might choose the second drug since the concentration in the bloodstream decreases so quickly.



Problem 2:

As described in problem 1, students may solve this problem either by substituting larger and larger values of t or they may use a graphing calculator to graph the function and determine the end behavior. A third option would be to consider properties of the exponential function:

> As t gets large, the graph of $y = e^{-t}$ will approach 0 (look at the graph's right-hand side). That means that the larger the value of t is, the closer to 0 the quantity 7.43 $e^{(-0.039\ t)}$ will be. Thus, as t gets large, the denominator gets closer to 1, so the value of P(t) approaches 1030437.

In terms of the problem, this means that the population of Shelby County is expected to "top out" at 1,030,437 if the trend established by the previous censuses continues.

Tasks for Student Work

A biology class performs an experiment comparing the quantity of food consumed by a certain kind of moth with the quantity supplied. The model for the experimental data is:

$$C(x) = \frac{1.648x - 0.002}{5.954x + 1}, \qquad x > 0$$

where x represents the quantity (in milligrams) of food supplied and y is the quantity (in milligrams) of food consumed. At what level of consumption will the moth become satiated?

An organization dedicated to preserving endangered species released 100 prairie chickens in a game preserve. The preserve has a carrying capacity of 1000 prairie chickens. The growth of the flock is modeled by: $p(t) = \dfrac{1000}{1 + 8.75e^{-0.1765t}}$, where t is measured in months.

a) Estimate the population after 6 months.
b) How long will it take the population to reach 500?
c) Determine the horizontal asymptotes, and interpret the horizontal asymptotes within the context of the problem.

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