PBS Lesson Series

Math: Grade 3, Lesson 17, Multiplication Facts and Strategies

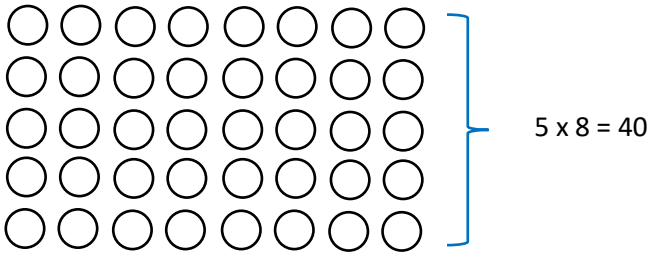
Lesson Focus: Use the distributive property to find products.
Practice Focus: Students will focus on practicing using the distributive property to break apart arrays in order to find products.
Objective: Students will break apart arrays to find products.
Key Vocabulary: factor, product, distributive property, array, partition
TN Standards: 3.OA.B.5
Teacher Materials:

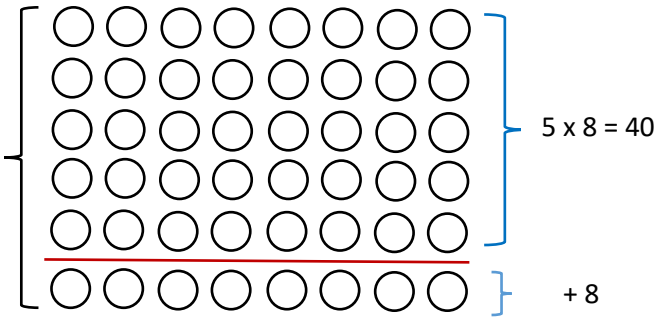
- Paper, pencil, and dry erase board/marker
- Counters or square tiles to build arrays
- Student Practice Packet

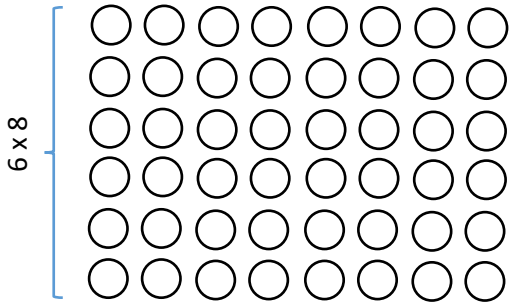
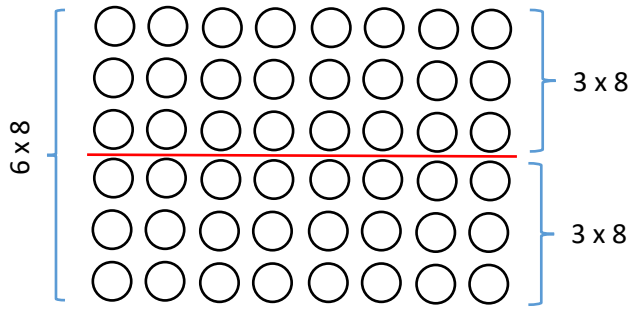
Student Materials:

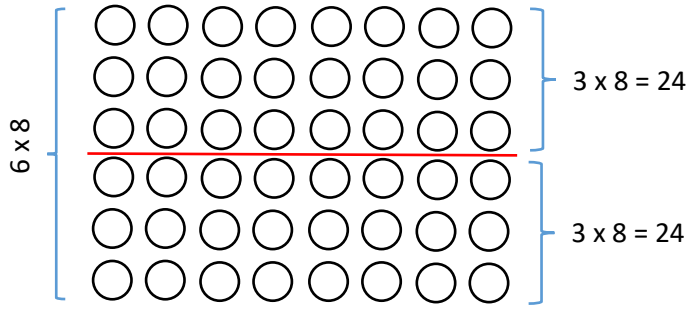
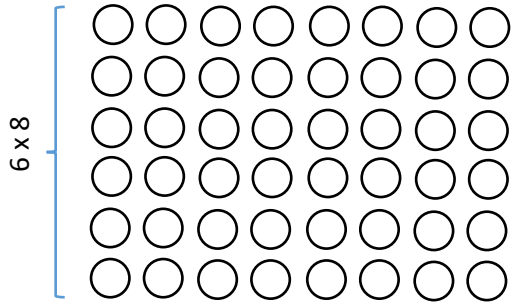
- Paper and a pencil, and a surface to write on

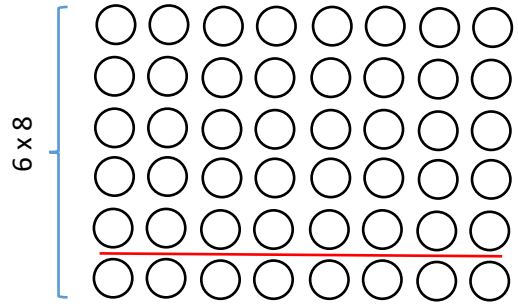
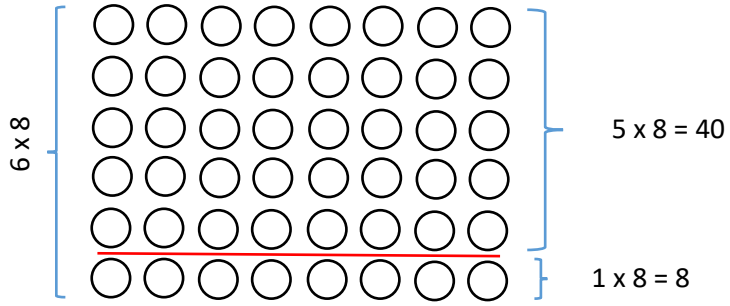
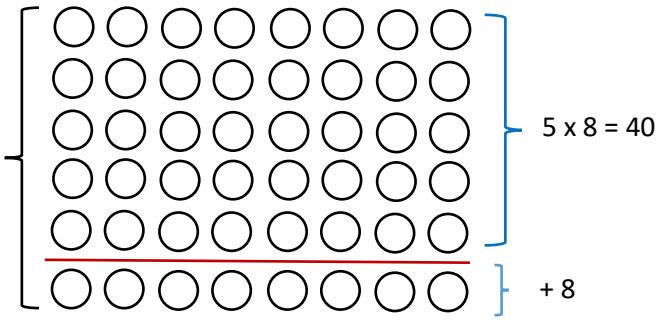
Teacher Do	Student Do
Opening (1 min) **Hello! Welcome to Tennessee's At Home Learning Series for math! Today's lesson is for all our 3rd graders out there, though all children are welcome to tune in. This lesson is the seventeenth in our series.** **My name is ____ and I'm a ____ grade teacher in Tennessee schools! I'm so excited to be your teacher for this lesson! Welcome to my virtual classroom!** **If you didn't see our previous lesson, you can find it on the TN Department of Education's website at www.tn.gov/education. You can still tune in to today's lesson if you haven't see any of our others. But, it might be more fun if you first go back and watch our other lessons since we'll be talking about things we learned previously.** **Today we will be learning about breaking apart arrays to find products in mathematics! Before we get started, to participate fully in our lesson today, you will need:** • **Paper and a pencil, and a surface to write on** • **The student packet for Math, Grade 3, Lesson 17 which can be found at www.tn.gov/education.** **Ok, let's begin!**	Students get materials ready for the lesson.
Intro (5 min) **If you were able to see the previous lesson, then you practiced using strategies to multiply by the factor 6. If you weren't able to join us for the previous lesson, that's okay because we're going to do a warm-up problem that is a**	

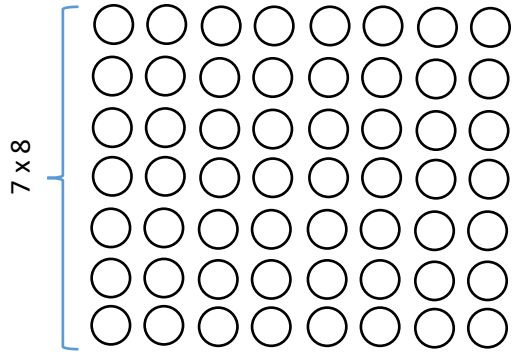
review. Then we'll practice a related strategy today that can help us find any product.	
For our warm up, let's use strategies to solve the multiplication equation 6 x 8 = ___. [Teacher records equation.] **Record this equation on your paper. There is a blank on the right side of the equal sign because we want to find the product for the factors 6 times 8.** [Teacher points to the factors and the blank line for product so students connect the vocabulary.]	Students record 6 x 8 = ___ on their paper.
We have learned that we can use known facts to find unknown facts. Specifically to find the product of 6 x 8, let's use the strategy you've practiced before called using a fives fact and addition. If 6 x 8 is an unknown fact, then we can use a known fives fact to help us find the product. That is, if we know the product for 5 groups of 8, we can use this fact to find the product for 6 groups of 8. Give me a thumbs up if that makes sense to you. [Pause.]	Students give a thumbs up to indicate understanding of the strategy using a fives fact and addition.
Let's draw a picture to help us visualize the strategy we're using. On your paper, draw an array to illustrate the fives fact 5 x 8 = 40. Show me with your fingers how many rows we'll have in our array. [Pause.] **Yes, we'll have five rows. Each row will have 8 in it. I'll use counters to build the array. You draw the array on your paper using dots. We'll compare our arrays in a minute.** [Teacher builds array as students draw array.]	Students prepare to draw an array to illustrate 5 x 8 = 40. Students show 5 fingers to indicate the number of rows in the array. Students use dots to draw the array for 5 x 8 = 40.
	
Check to see that your array has 5 rows of 8. We know that 5 x 8 = 40. Record this equation next to your array. [Teacher records equation next to her array.]	Students check their arrays and record the multiplication equation.
Now, instead of drawing a separate array to find the product of 6 x 8, how can we use this array? [Teacher points to the 5 by 8 array.] **The strategy we're using says to use a fives fact and addition. We have our fives fact as an array and as an equation. What do you think our next move is? Say your answer out loud.** [Pause.]	Students verbalize that the next step is to draw another row of 8.

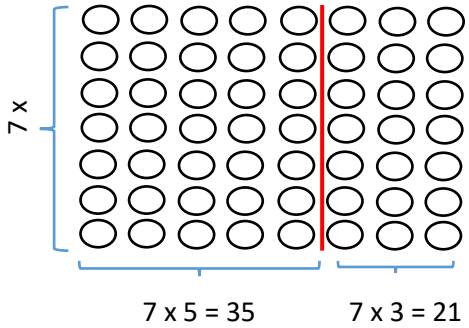
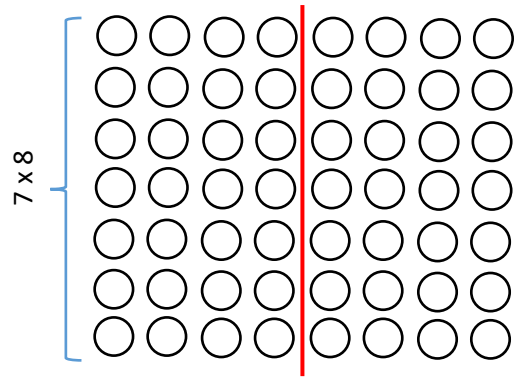
Give me a thumbs up if you said we can just add another row of 8 to this array to find the product of 6 x 8. [Pause.] **Great! Let's illustrate adding another row of 8 to our array. So we can see the parts of the whole array, draw a line under the 5 by 8 array and then draw another row of 8 under the line. I'll do the same with my counters.** [Teacher draws a line under the 5 by 8 array, and then adds another row of 8 so array looks like image below.]	Students draw a line under their 5 by 8 array and then draw another row of 8.
	
After you draw the additional row of 8, label it with the expression + 8. [Teacher points to her array label.] **Now we have composed the parts 5 rows of 8 and an additional row of 8 to make a larger array. Use your fingers to show me how many rows of 8 we now have on the whole array.** [Pause.] **Yes, we now have 6 rows of 8 so we have what we need to find the product of 6 x 8. Using your picture as your guide, complete the equation 6 x 8 = ___ + ___ = ___. Record the completed equation on your paper below your drawing.** [Pause.]	Students label their extra row with the expression + 8. Students show six fingers to indicate the total number of rows. Students complete the equation 6 x 8 = ___ + ___ = ___.
Our completed equation is 6 x 8 = 40 + 8 = 48. [Teacher records equation below her 6 by 8 array.] **Give yourself some applause for completing this equation! Do you see where the expression 40 + 8 is in your drawing?** [Teacher points to her representation of 40 + 8.] **The 40 is the fives fact and the + 8 is the additional row of 8. Those two parts added together give us the total for the whole array which is 6 x 8 = 48. We just used the strategy use a fives fact and addition to find the product of 6 x 8. We drew a picture and we wrote an equation to illustrate the strategy.** [Pause.]	Students check equation is 6 x 8 = 40 + 8 = 48 and give themselves applause.
Great job reviewing the strategy of using five facts and addition to multiply by 6. This strategy is related to the strategy we'll practice today which is using the distributive property to break apart arrays to find products.	

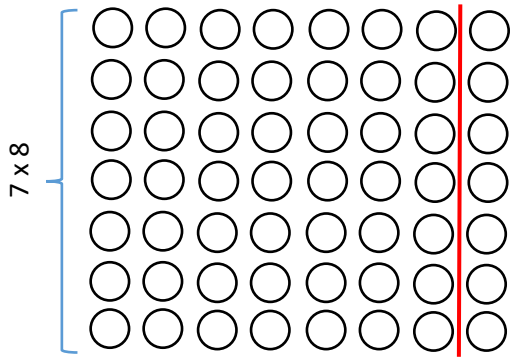
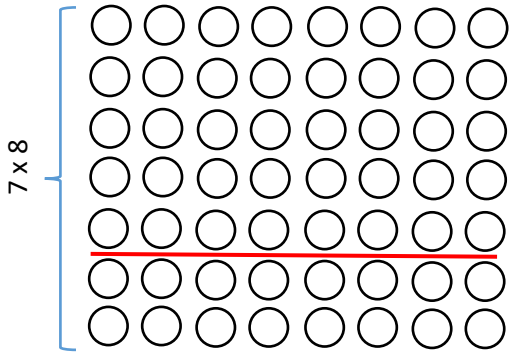
Teacher Model (10 min) Objective 1: Teacher modeling using the distributive property to break apart arrays (number of rows) into two smaller arrays whose sum of their products equals the product of the whole array. **Let's revisit the multiplication expression 6 x 8. In our warm-up problem, we started with a 5 by 8 array and then added another row to build a 6 by 8 array. What if we already have a drawing of a 6 by 8 array? Instead of counting the total counters or dots to find the product of 6 x 8, can we partition or cut the whole array into two smaller arrays whose products we can add together to find the product of the whole array? What do you think?** [Pause.] **Let's find out. On your paper, use dots to draw a 6 by 8 array.** [Teacher builds or draws 6 x 8 array like below image.]	Objective 1: Students will be reviewing breaking apart arrays into smaller arrays as a strategy to find the product of a multiplication fact. Students will have another exposure to the term distributive property. This will support students developing multiplication fact fluency. Students consider if breaking apart an array into two smaller arrays can help them find the product of the whole array.
	Students use dots to draw the 6 by 8 array.
Let's draw a line to break our 6 rows of 8 into two smaller arrays. Break your array apart like mine this time. Draw a horizontal line so that you have two arrays that both have 3 rows of 8. [Teacher draws line to break apart array like image below.]	Students draw a horizontal line to break apart their 6 by 8 array into two smaller arrays of 3 x 8 and 3 x 8.
	
Give me a thumbs up if your picture looks like mine. Be sure you have labeled the whole array and the two smaller arrays. [Pause.] **Great! We started with an array matching the multiplication expression 6 x 8. We still have that whole**	Students give a thumbs up to indicate they've checked their drawing.

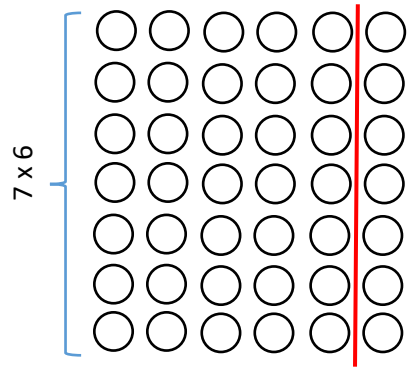
array, but we also now have it broken apart into the two smaller arrays expressed as 3 x 8 and 3 x 8. You are really fluent with your threes facts, so say out loud the product of 3 x 8. [Pause.] **Yes, 3 x 8 = 24. Show this product for the smaller arrays on your drawing.** [Teacher displays her array and changes 3 x 8 to an equation.]	Students verbalize 3 x 8 = 24. Students label the smaller arrays with 3 x 8 = 24.
	
We know the product of each of the smaller arrays is 24. How can we use these products to find the product for the whole array? [Pause.] **When we broke apart the array, we didn't add or remove any rows or counters. The total number of counters is equal to the sum of its parts. On your paper, complete the equation 6 x 8 = ___ + ___ = ___.** [Pause.]	Students consider how this can help them find the product of the whole array. Students record and complete the equation 6 x 8 = 24 + 24 = 48.
Our completed equation is 6 x 8 = 24 + 24 = 48. We just illustrated the strategy we're going to practice today! Our strategy to find a product is to break apart an array into smaller arrays and then find the sum of their products. This strategy is also known as the distributive property. **One important thing to know is that there are many ways to break apart an array. Let's look at the 6 by 8 array again as a whole.** [Teacher displays array.]	
	
If I know that I'm really fluent with my fives facts, I might decide to break apart my array like this. [Teacher shows array broken apart.]	Students observe teacher breaking apart the 6 by 8 array in a different way.

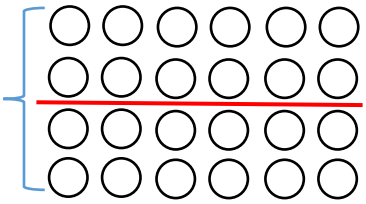
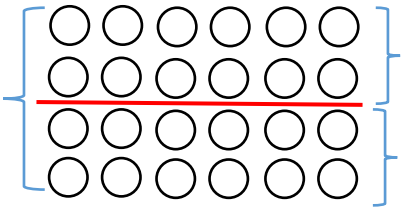
 **On your paper, write the multiplication equations for my two smaller arrays.** [Pause.] **For the array with 5 rows of 8, the multiplication equation is 5 x 8 = 40. For the array with 1 row of 8, the multiplication equation is 1 x 8 = 8.** [Teacher labels her arrays as below.]	Students write the multiplication equations that describe the two smaller arrays. 5 x 8 = 40 and 1 x 8 = 8.
 **Now complete this equation on your paper.** [Teacher records the equation 6 x 8 = ___ + ___ = ___.] **The completed equation is 6 x 8 = 40 + 8 = 48. Again, we find 6 x 8 = 48 by breaking the whole array into two smaller arrays and then adding their products together. The total number of counters never changed. Because we drew a horizontal line to break apart the array, the total number in each row never changed, only the number of rows changed.**	Students complete the equation that describes the parts and whole of the array. 6 x 8 = 40 + 8 = 48.
Did you notice how cutting the array into 5 rows and 1 row resembled the strategy we used in the warm-up problem? [Pause.] **This was our drawing when we showed the strategy of using a fives fact and addition.** [Teacher shows drawing below.] 	Students connect breaking apart the array into 5 rows and 1 row to the strategy using a fives fact and addition.

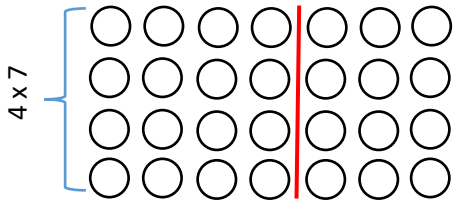
The whole array has the same parts, but we did not write the additional row of 8 as the multiplication equation 1 x 8 = 8. Also, in the five facts and addition strategy, we built the parts first in order to make the whole array. **In the strategy using the distributive property, we begin with the whole array and break it into smaller arrays of our choosing. So far we have used the distributive property strategy to break apart the 6 by 8 array into smaller arrays that each have fewer rows but the same number of counters inside each row. We can also use this strategy to find products by breaking apart arrays using a vertical line. Let's try this now to solve the multiplication problem 7 x 8 = ___.**	
Objective 2: Teacher modeling using the distributive property to break apart arrays (number within each row) into two smaller arrays whose sum of their products equals the product of the whole array. **To find the product for 7 x 8, first let's draw the array. On your paper, use dots to draw seven rows of eight.** [Teacher draws array below.] 	Objective #2: Students will be reviewing breaking apart arrays into smaller arrays as a strategy to find the product of a multiplication fact. Students will see that they can draw a vertical or horizontal line to make the partition. Students use dots to draw the 7 by 8 array on their paper.
Draw a vertical line to break apart your array so that one array has 5 counters inside each row and the other array has 3 counters inside each row. Label the smaller arrays with a multiplication equation. [Teacher partitions and labels array as image below.]	Students draw a vertical line to break apart the array so that one array has 5 counters per row and the other array has 3 counters per row.

 **Check to see that your drawing is similar to mine and that you have labeled the two smaller arrays with the appropriate multiplication equations. Notice that drawing a vertical line doesn't change the number of rows in each array. Instead it breaks apart the number of items in each row. Now that we have our two smaller arrays, what is our next step in order to find the product of the whole array?** [Pause.] **Give me a thumbs up if you said that our next step is to add together the two products from the smaller arrays. Great thinking! On your paper, complete the equation 7 x 8 = ___ + ___ = ___.** [Pause.]	Students check their drawing and labels to match teacher's work. Students give a thumbs up to indicate they said the next step is to add together the products from the smaller arrays.
The completed equation is 7 x 8 = 35 + 21 = 56. Check to see that your equation is the same. [Pause.] **The expression 35 + 21 shows that we are adding the two parts together to get the whole.**	Students check that their completed equation is 7 x 8 = 35 + 21 = 56.
Tying the learning together: Students understand that there are many ways to break apart arrays into smaller arrays, and the sum of the smaller arrays equals the total of the whole array. **Let's think about some other ways we could have broken apart the 7 by 8 array. Let me show you the whole array again.** [Teacher displays 7 by 8 array.]	Tying the learning together: Students connect the idea that arrays can be broken apart in many ways to the idea that the sum of the parts equals the whole.
Visualize another way to break apart the array with a vertical line so that there are two smaller arrays. On your paper, write the multiplication equations for these arrays. [Pause.] 	Students visualize another way they could have broken apart the 7 by 8 array with a vertical line. They record the matching multiplication equations on their paper.

[Teacher shows image above.] **If you broke apart the array with a vertical line here, the two multiplication equations are 7 x 4 = 28 and 7 x 4 = 28. The sum of the two parts, 28 + 28, equals 56.**  [Teacher shows image above.] **If you broke apart the array with a vertical line here, the two multiplication equations are 7 x 7 = 49 and 7 x 1 = 7. The sum of the two parts, 49 + 7, equals 56.** **I hope you noticed that when we broke apart the array using a vertical line that the first factor in the equations stayed the same, in this case 7. That's because the whole array and the smaller parts all had 7 rows. We broke apart the number in the rows which is expressed by the second factors.** **Let's look at the 7 by 8 array one more time. Visualize breaking apart the array with a horizontal line this time. Write the two multiplication equations that represent the two smaller arrays.** [Pause.]  	Students listen to teacher describe other ways the array could be broken apart. Students see that regardless of the partition, the sum of the products is always 56. Students visualize another way they could have broken apart the 7 by 8 array with a horizontal line. They record the matching multiplication equations on their paper.

[Teacher shows image above.] **If you broke apart the array with a horizontal line here, the two multiplication equations are 5 x 8 = 40 and 2 x 8 = 16. The sum of the two parts, 40 + 8, equals 56.** **If you broke apart the array a different way, check to see that the sum of your two products is equal to 56!** **I hope you noticed that it didn't matter if we broke apart the array with a vertical line or a horizontal line, we still were able to find the product of the whole array by adding together the products from the smaller arrays. The total number of counters stay the same, what changes is the way we distribute the counters. This is what it means to use the distributive property. That is, the product of the whole array is equal to the sum of the products from the two smaller arrays.**	Students check to see that the sum of their parts is equal to the whole which is 56.
Guided Practice (10 min) **Let's practice finding products using the distributive property!** [I do.] **I'll do the first practice problem.** [Teacher reads the problem out loud and does a think aloud as she solves it.] **The array 7 x 6 has been broken into two smaller arrays. Write the multiplication equations that describe the smaller arrays. Use the smaller arrays to find the product of 7 x 6.** [Teacher shows array below.]  	Students listen to teacher solving a problem using the distributive property to find a product.

[Teacher does think aloud.] **I see that the 7 by 6 array has been broken apart with a vertical line. Both of the two smaller arrays have 7 rows like the whole array. The number of counters in the rows of one array is 5 and in the rows of the other array is 1. This makes sense because in the whole array, there are 6 counters in each row.** **The multiplication equations of the smaller arrays are 7 x 5 = 35 and 7 x 1 = 7. I can add these products together to get the total number of counters in the whole array. So, 7 x 6 = 35 + 7 = 42. The product of 7 x 6 = 42.**	
[We do.] **Now let's do this next problem together.**	Students practice solving a problem with the teacher.
The array 4 x 6 has been broken into two smaller arrays. Write the multiplication equations that describe the smaller arrays. Use the smaller arrays to find the product of 4 x 6. [Teacher shows array below.] 4 x 6 	
Use dots to make a quick drawing of this array on your paper. You will have 4 rows of 6 dots. [Pause.] **Label your drawing like this array so that the multiplication expression 4 x 6 is to the left of the array. And draw the horizontal line that breaks apart the array.** [Pause.]	Students use dots to draw a quick array of 4 x 6. Students draw a horizontal line to match the teacher's work.
Look at the smaller arrays. They are the same size. Show me with your fingers how many rows are in one of the smaller arrays. [Pause.] **Yes! There are two rows in the smaller arrays. On the right side of each smaller array, write its multiplication equation.** [Pause.]	Students show 2 fingers to indicate the smaller arrays have 2 rows.
4 x 6  2 x 6 = 12 2 x 6 = 12	Students record the equations 2 x 6= 12 and 2 x 6 = 12 next to the smaller arrays on their drawing.

Check to see that your drawing and equations match this picture. [Pause.]	Students check their drawings and their equations.
We've written the multiplication equations that describe the smaller arrays. How do we use these facts to find the product of 4 x 6? [Pause.] **Complete this equation to show your thinking. 4 x 6 = ___ + ___ = ___.** [Pause.]	Students complete the equation 4 x 6 = 12 + 12 = 24.
Read out loud with me the completed equation. **4 x 6 = 12 + 12 = 24.**	Students read the equation out loud with teacher.
In this completed equation, circle the expression that indicates the smaller arrays. [Pause.] **Give me a thumbs up if you circled the expression 12 + 12. Great job! Now underline the expression that is the product of the whole array, or 4 x 6.** [Pause.] **Give yourself some applause if you underlined 24 for the product of the whole array.**	Students circle the expression 12 + 12 in their equation because this is the part that refers to the products of the smaller arrays. Students give themselves applause for underlining 24 as the product of the whole array.
[You do.] **Now you try one by yourself! Refer to your notes if you need to!**	
[Teacher reads the problem and displays the array below.] **The array 4 x 7 has been broken into two smaller arrays. Write the multiplication equations that describe the smaller arrays. Use the smaller arrays to find the product of 4 x 7.**  	Students solve a problem alone using the products of the smaller arrays to find the product of the whole array.
[Teacher allows students time to solve before sharing answer.]	
The multiplication equation that describes the smaller array on the left is 4 x 4 = 16. The multiplication equation that describes the smaller array on the right is 4 x 3 = 12. Check your equations. [Pause.]	Students check their work to make sure they recorded 4 x 4 = 16 and 4 x 3 = 12 for the two smaller arrays.
The second part of the problem asks you to use the smaller arrays to find the product of 4 x 7. You may have written an equation like this: 4 x 7 = 16 + 12 = 28. Check your work to make sure you found that the product of 4 x 7 = 28.	Students check their final equation to make sure they show 16 + 12 = 28 and that the product of 4 x 7 is 28.

Great job! Additional Problems (if needed): A marching band has 4 rows of trumpeters with 10 trumpeters in each row. How many trumpeters are in the marching band? Use the Distributive Property to solve. What are some ways you could break apart 7 x 9 using the Distributive Property?	
Independent Practice (10 min) **Great work, students! Today, we reviewed using the distributive property to find products. That is, we broke apart an array into smaller arrays and then added their products to find the product of the whole array. I hope you are feeling confident that you can use known facts to find unknown facts. You sure did a great job! I will show you the independent practice problems now, or you can find them in the student practice for this lesson posted on our website, www.tn.gov/education.** [Teacher shows student practice page under document camera or camera zooms in on student practice page.] **Good luck and do your best!**	Students listen to teacher summarize today's learning and view the independent practice problems.
Closing (1 min) **Students, I enjoyed practicing using the distributive property to find products of multiplication facts! Thank you for inviting me into your home. I look forward to seeing you in our next lesson in Tennessee's At Home Learning Series! Bye!**	