Math Fundamentals for Operators in Training

Course # 1009 or 1009-V

Math Fundamentals for Operators in Training
February 23-25, 2022
Course #1009 or 1009-V

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Wednesday
8:30	Registration and Welcome
8:45	Numbers and Operations
10:00	Fractions
11:00	Lunch
12:15	Decimals and Percentages
2:00	Powers, Roots, and Scientific Notation

Thursday
8:30	Review Day 1
9:00	Order of Operations
10:00	Introduction to Equations
1100	Lunch
12:15	Ratios and Proportions
1:30	Conversions and Dimensional Analysis

Friday
8:30	Exam Review & Practice
11:15	Lunch
12:30	Exam

Fleming Training Center
Your Partner in Clean Water

TN Department of Environment & Conservation

Math Fundamentals
Table of Contents

Section 1

Numbers and Operations

SOLVING MATH PROBLEMS

Chapter 1
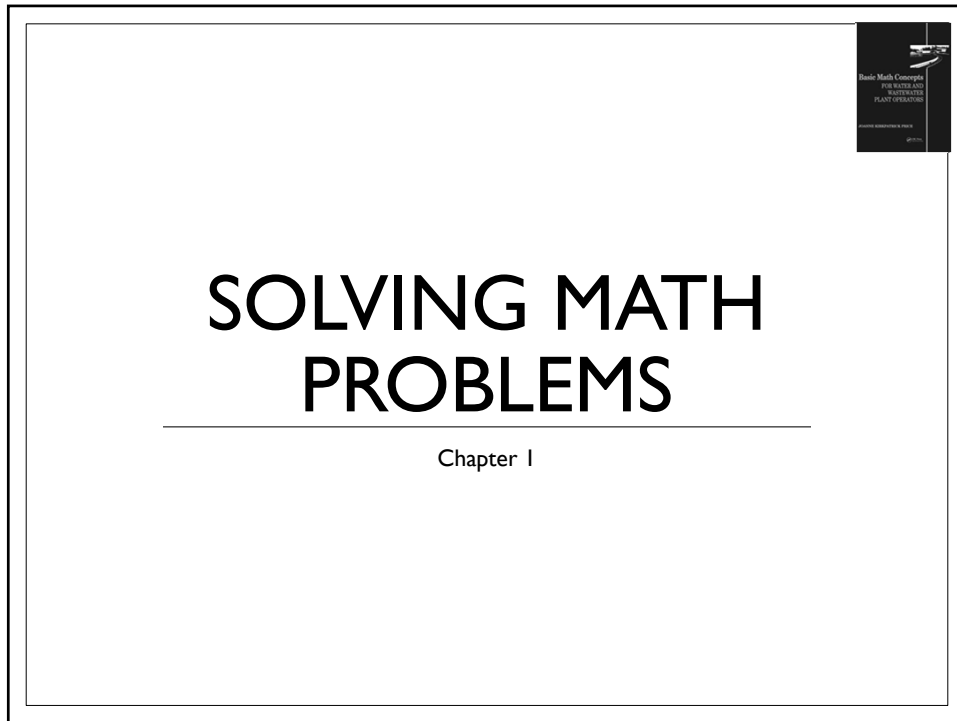
Difficulties in Math

- Poor Foundation
  - Mathematics is sequential.
    - Solution: Find your "weak spots" and focus on strengthening.

- No Linking or Steps Missing
  - New concepts must be linked to a concept you already know to create a "mental chain;" without that linking, remembering a new concept is difficult.
    - Solution: Be sure new material make sense and connects with what you already know. If you are struggling, ask for help.

Difficulties in Math

- The "Big Picture" is Missing
  - Solution: Concentrated effort must be made to include the underlying concepts for topics learned.

- "Use It or Lose It" Syndrome
  - The more you practice & use various math calculations, the easier they become.
    - Solution: Use math whenever possible, not only when studying for the exam

Theoretical vs Applied Math

- *Theoretical math* includes the "tools" of math or the math concepts (fractions, decimals, percents, areas, volumes).
  - Math done for its own sake.
  - Includes those topics covered in this class.

- *Applied math* is basic math concepts applied in solving practical problems.
  - Math that has a practical use.
  - Covered in the two following weeks of Applied Math classes.

Solving Math Problems

- Suggested Strategies
  - Disregarding all numbers, what type of problem is it asking you to find?
  - What diagram, if any, is associated with the concept being questioned?
  - What information is required to solve the problem and how is it expressed in the statement of the problem?
  - What is the final answer to the problem?
  - Does the answer make sense?

NUMBERS AND OPERATIONS

California State University: Sacramento

Water Treatment Plant Operation, vol. 1

Appendix A.1

Basic Concepts

- An *integer* is a whole number used to count things.
  - Example: 2   10   789

- A *fraction* is ratio of two integers.
  - Example: $^1/_4$ $^8/_5$ $^{24}/_{897}$
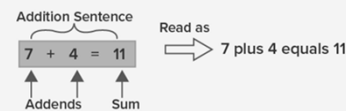
- A *decimal* is a shorthand way of expressing an integer plus a fraction.
  - Example: $10.3 = 10 + (^3/_{10})$
  - Represents whole numbers and parts of the whole.
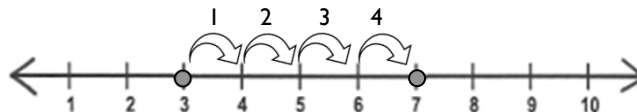 - Example: 0.25   1.6   3.0267

Addition

- *Addition* is bringing two or more numbers (or things) together to make a new total.

- When adding numbers (*addends*), the end number (*sum*) will be higher than the addends.
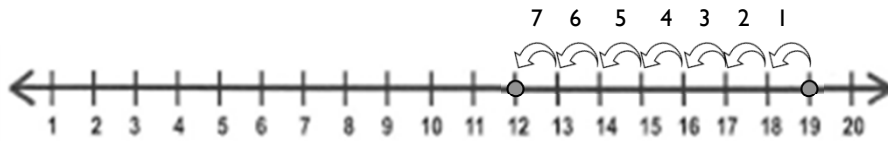
- Find the sum of 3 plus 4.

$$3 + 4 = ?$$

$$3 + 4 = 7$$

Subtraction

- *Subtraction* represents removing objects from a collection.
- When subtracting numbers, the *difference* between the numbers will be less than the numbers.
- Find the difference between 19 and 7.

$$19 - 7 = ?$$

$$19 - 7 = 12$$

Multiplication

- *Multiplication* gives the result of combining groups of equal sizes.
- A *factor* is a number that is multiplied by another number to find a product.
- The answer in a multiplication problem is called the *product*.
  - Can be found by multiplying the factors.
- Distributive Property of Multiplication
  - The multiplication of whole numbers may be thought of as a repeated addition.
  - Example:

$$15 \times 3 = 45$$

$$15 + 15 + 15 = 45 \qquad \text{OR} \quad (10 \times 3) + (5 \times 3) = 45$$

Multiplication - Example

- A system uses 5 gallons of chemical a day. How many gallons of chemical will the system use in 4 days?

5

+

5

+

5

+

5

5+5+5+5 = 20

OR
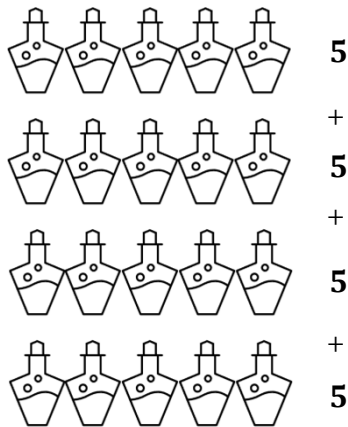
5 x 4 = 20

Division

- *Division* is the separation of a number into equal sized groups.
- The *dividend* is the number that is to be divided in a division problem.
  - aka Numerator
- The *divisor* is the number that divides the dividend.
  - aka Denominator
- The *quotient* is the number, not including the remainder, that results from division.

$$18 \div 3 = 6$$

Dividend   divisor   quotient

Division - Example

- A system feeds 20 gallons of chemicals over a 4-day period. How many gallons are fed each day?
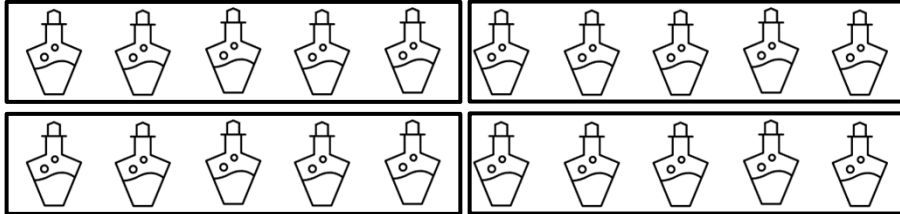
- If we split up 20 gallons into 4 equal groups, how many gallons are in each group? 5

$$20 \div 4 = 5 \qquad\qquad \frac{20}{4} = 5$$

Math Fundamentals
Numbers and Operations
Practice Problems

1. There are 38 books on the shelf. Marta put 10 more books on the shelf and her mom put 4 more books on the shelf. How many books are on the shelf now?

2. Tim had some peanuts. He gave 40 peanuts to Steve and 30 peanuts to Jim. Now he has 20 peanuts left. How many peanuts did he start with?

3. I have 28 pennies and 3 rocks in my left pocket. In my right pocket I have 6 pennies, 2 rocks and 4 candies. How many pennies do I have? How many rocks?

4. Josh had 16 marbles in his collection. He lost 7 marbles. How many marbles does he have now?

5. Megan has 19 seashells. How many more seashells does she need to find to have 25 seashells in her collection?

6. Zach scored 42 points in the football game. Ben scored 21 points. How many more points did Zach score?

7. Brad has 17 balloons. 8 balloons are red, and the rest are green. How many green balloons does Brad have?

8. A single bee has 6 legs. How many legs do 8 bees have?

9. Jeff has twice as many crayons as Joan. Joan has 12 crayons and 6 pencils. How many crayons does Jeff have?

10. Mrs. Scarlet bought a new hat for $15. At the same price, how much would 3 hats cost?

11. Mrs. White spent 10 minutes shoveling snow each day this week. How many minutes time did she spend shoveling snow?

12. James works 5 shifts each week at a local restaurant. If James made $475.00 this week, what is his average daily wage?

13. A backhoe uses 144 gallons of diesel over a three (3) day period. The takes 1.5 months to complete. How many gallons of diesel are used up in an average day?

14. Charlie and his father, an engineer, decided to build a treehouse in their backyard.
    a. In order to start constructing the house, Charlie and his father needed to gather some wood from the forest. If they initially have 15 extra planks of wood in the house and Charlie and his father got 10 planks of wood each, how many pieces of wood do they have in total?

b. While building the house, Charlie noticed that they were running out of nails, so he told his father he is going to buy some. If they still have 9 nails left and Charlie bought 2 boxes of nails, the big one containing 55 nails and the small one containing 31, how many nails will they have?

c. To have a more stable treehouse, Charlie's father decided to tie the corner posts of the house to the tree itself. He used 24 inches of rope for the first post, 20 inches on the second, 14 inches on the third and 12 inches on the fourth. He also had 15 feet of cable but did not use that. How many inches of rope were used?

d. The treehouse is almost done; all they need is to paint it. His father estimated that they will use 20 ounces of white paint, 15 ounces of green paint and 34 ounces of brown paint and 6 paintbrushes. How many ounces of paint would they buy in total?

e. Upon finishing the treehouse, Charlie's mother served them freshly baked cookies. If Charlie ate 15 cookies, his father ate 10 and his mother only ate 5, please write an equation to show how many cookies were eaten in total?

15. Winter is coming, and most animals are migrating to warmer countries.
   a. There are 67 bird families living near the mountain. If 32 bird families flew away for winter, how many bird families were left near the mountain?

   b. At the river, 25 out of 55 salmon families went to warmer waters to avoid being frozen. They had to swim 125 miles to get there! How many salmon families were left in the river?

c. The wolves, though accustomed to cold weather, also wanted to move away from the incoming winter. If there are 43 packs of wolves living in the forest and 31 packs went away, how many wolf packs were left in the forest?

d. Some deer families are also moving out to avoid the shortage of grass that will result from the snow. If there are 79 deer families in the area and 45 of them stayed, how many deer families moved out?

e. Even the chipmunks tried to get away to find other warmer places to stay. They would have to walk for 14 days to get somewhere warmer. If 21 chipmunks were left from the original 86, how many chipmunks went away?

16. It was the Crazy Sales Week at the department store. Any item that is $2,000 or more will get an instant discount of $200. People buying two pieces of clothing items gets $15 off the second item.
    a. Josh wants to buy a TV that costs $2,380 and a printer that costs $259. How much does he need to pay?

    b. Ashley finds a dress that costs $116 and a hat that costs $42. How much does she need to pay?

    c. Sean wants to buy a laptop computer that costs $2,099, a shirt that costs $22 and a jacket that costs $136. How much does he need to pay?

    d. Among the three persons, who spent the most money at the store?

e. On Monday, 126 laptops computer were sold. On Tuesday, 68 laptop computers were sold. On Wednesday, 283 laptop computers were sold. How many computers were sold in the three days?

17. Andrew is having his friends over for game night. So, he decided to prepare snacks and games.
    a. He started by making mini sandwiches. If he has 4 friends coming over and he made 3 sandwiches for each one of them, how many sandwiches did he make?

    b. He also made some juice from fresh oranges. If he used 2 oranges per glass of juice and he made 6 glasses of juice, how many oranges did he use?

    c. Then he started to prepare the games for his 4 friends. If each game takes 5 minutes to prepare and he prepared a total of 5 games, how many minutes did it take for Andrew to prepare all the games?

    d. Andrew's 4 friends decided to bring food as well. If each of them brought 4 slices of pizza and 3 bags of chips, how many slices of pizza do they have in total?

    e. Lasty, Andrew tried to compute his expenses for the game night. If he spent $9 for each game they played, and they played a total of 5 games. How much money did he spend on games that night?

18. Lexie has a lot of art materials. She needs to organize all these materials into containers.
    a. She counted her crayons and found out that she has 80 crayons which she will place in crayon boxes. Every crayon box can contain 8 crayons. How many crayon boxes does she need?

b. 3 piles of clean white papers were stacked in the corner of her room. She decided to place these papers in paper envelopes which can hold 10 papers each. How many paper envelopes does she need if she has 120 clean white papers?

c. Besides the piles of white paper was a stack of 700 sheets of used paper. She wants to place it in boxes for recycling. If every box can contain 100 sheets, how many boxes does she need?

d. Lexie's spent 2 hours gathering all of her watercolor paintings. She thought of placing an equal number of paintings in four of the rooms in the house. If Lexie has 32 watercolor paintings, how many paintings will be placed in each of the four rooms?

e. Lexie's younger brother helped pick up all the paper clips in Lexie's room. He was able to collect 81 paper clips. If he wants to distribute the paper clips in 9 boxes, how many paperclips each box will contain?

19. A team of 24 customer service representatives works at a customer service hotline which operates 24 hours a day. There are four shifts. Normal shifts are from 5 am to 11 am, 11 am to 5 pm and 5 pm to 11 pm. Midnight shift is from 11pm to 5am. On average, 630 calls are received every day during the 3 normal shifts and 115 calls are received during the midnight shift.

a. If only 3 customer representatives work during the midnight shift, how many representatives work on each of the other three shifts?

b. On average, are there more calls received during one normal shift or one midnight shift?

   c. On average, how many calls are received by each representative during each hour of the normal shift?

   d. The representatives that work on normal shifts will be paid $12 per hour and those who work on midnight shift will be paid $18 per hour. What is the daily salary for a representative that works the midnight shift?

20. During a normal day, there are 280 planes taking off from the airport, but the airport is a lot busier during Christmas. During the Christmas holidays, about 336 planes take off every day from the airport.

   a. During the Christmas holidays, the airport opens 12 hours during each day, how many planes take off from this airport in each hour?

   b. In average, each plane takes 240 passengers and 12 tons of cargo. How many passengers depart from the airport every hour during the Christmas holidays?

   c. Compared with a normal day, how many more passengers depart from the airport in a day during the Christmas holidays?

   d. During a normal day, there are 782 passengers in average that are late for their plane each day. However, during the Christmas holidays, there are 1,835 passengers that are late for their planes each day which caused delays of 14 planes. How many more passengers are late for their planes in each day during the Christmas holidays?

e. The airport administration did a study and found that an additional 5 minutes of delay in the overall operation of the airport is caused for every 32 passengers that are late for their flights. What is the delay in the overall operation if there are 832 passengers late for their flights?

21. The library is being relocated. The librarians are busy organizing the books.
   a. Three boxes filled with fiction books and two boxes filled with reference books were just delivered. If each box is filled with 120 books, how many fiction books are there?

   b. From the boxes just delivered, a librarian takes out 40 reference books to put on the shelf. Then, she takes out another 65 reference books and leave them at the checkout counter. How many reference books are left in the boxes?

   c. There were 445 non-fiction books but 83 went missing and 45 were transferred to another library. How many non-fiction books are left?

   d. According to the library system, there are 238 new patrons, 145 inactive patrons and 673 active patrons. How many patrons are there in total in the library system?

   e. There are 2,475 books in total in the library. 592 books are on loan and 137 books are missing. Out of the 592 books, 74 books are late. How many books are there in the library?

22. There are 4 doctors working in a clinic. Each doctor has 2 nurses assisting them. There are two receptionists, Jay, and Molly, working at the reception.
    a. How many people are working in the clinic?

    b. On Monday, 23 patients made appointments with each doctor. However, 6 of the patients did not show up. How many patients visited the clinic on Monday?

    c. On Tuesday, Jay answered 45 phone calls and Molly answered 12 more calls than Jay did. How many calls were answered in total?

    d. On Wednesday, a doctor called in sick. The two nurses and Jay together called 36 patients to reschedule their appointments. They shared the calls evenly. How many calls did they each make?

    e. Among the calls they made, 13 of the patients decided to cancel their appointments and the rest decided to postpone their appointments. How many appointments were postponed?

23. A stadium has 10,500 seats and 8 VIP boxes. The stadium is divided into 12 equal sections: 2 premium sections and 10 standard sections. A seat at the premium section costs $48 per game. A seat at the standard section costs $27 per game.
    a. How many seats are there in each section?

    b. If there are 35 seats in each row, how many rows are in each section?

c. If all the seats in the premium section are sold out for a game, how much will the stadium get from those ticket sales?

d. There are 50 games in each season. A season pass costs $2,040. A season pass holder can go to all the games and have a seat in the premium section. How much can a fan save by buying the season pass?

e. For the night game on Tuesday, 8,395 tickets were sold. How many tickets were left?

24. A library has 3,489 non-fiction books, 8,617 fiction books and 1,240 reference books.
   a. All books, except the reference books, are available for loan. How many books are available for loan?

   b. Reference books are for use in the library. There are 16 bookshelves for the reference books. After use, they need to be returned to a special collection box for shelving. If 128 reference books are in use and 84 reference books are in the collection box, how many reference books are on the shelf?

   c. Each patron pays an annual fee of $36 to the library. If the library collects $20,304 from the annual fee, how many patrons are there?

   d. Each patron can borrow up to 6 books. If all the patrons are currently holding on to 6 books each, how many books are left in the library?
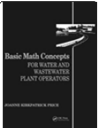
e. Each patron can borrow the books for 2 weeks and renew the loan twice. What is the maximum number of days can a patron keep the books he borrowed from the library?

25. Write an equation using "x" to represent the unknown value. There were 156 TV sets in storage. To prepare for Crazy Sales Week, the store ordered 470 TV sets. During the Crazy Sales week, x TV sets were sold, and 105 TV sets were left in the storage.

26. Write an equation using "F" to represent the unknown value. Andy hosted a dinner party. He can spend $105 on food for the party. If each meal costs $15, how many friends can Andy invite?

27. Write an equation using "R" to represent the unknown value. Midge bought souvenirs for 6 of her friends back home. If she wanted to get 3 souvenirs for each friend, how many souvenirs did Midge buy?

28. A local scout makes wood carvings as a hobby. The display shelves can each hold 8 carvings. If the scout has made 136 carvings, how many shelves will be needed to display all the pieces?

29. According to the customer representative guidelines, the representative should rest for x minutes for every hour of their duty. During each hour of their duty, they should answer 5 calls which means they have around 10 minutes of each call. Write an equation using "x" to represent the unknown value.

30. On the New Year Eve, there were 7,580 tons of cargo loaded in the morning. In the afternoon, there were x tons of cargos. The total weight of cargos loaded on the day weighed 12,997 tons. Write an equation using "x" to represent the unknown value.

31. A public library bought 115 new fictions, 38 non-fiction and x new reference books. They ordered 174 books in total. Write an equation using "x" to represent the unknown value.

32. For each patient they see, each doctor can get paid $80. If there are x patients on that day, the doctor will be paid $1,760. Write an equation using "P" to represent the unknown value.

33. A stadium has 10,500 general seats and 8 VIP boxes. If all the general seats and all the VIP boxes are filled, the stadium has 10,628 people. How many seats are in each VIP box? Write an equation using "V" to represent the unknown value.

34. A library patron had a number of overdue books. The late fee for each book is $2 for every day the book is late. The patron paid $72 for books that are 9 days overdue. How many books were checked out? Write an equation using "B" to represent the unknown value.

Numbers and Operations Answers

1.)	52	16.)	a. ,2839	21.)	a. 360	25.)	105
2.)	90		b. 173		b. 240	26.)	105 = 15 x F
3.)	34; 5		c. 2,472		c. 317	27.)	6 x 3 = R
4.)	9		d. 477		d. 1,056	28.)	X = 136 ÷ 8
5.)	6	17.)	a. 15		e. 1,746	29.)	60 = (5 x 10) + X
6.)	21		b. 18	22.)	a. 14	30.)	12997 = 7580 + X
7.)	9		c. 25		b. 86	31.)	174 = 115 + 38 + X
8.)	48		d. 16		c. 102	32.)	1760 = 80 x P
9.)	24		e. 45		d. 12	33.)	10628 – 10500 = V x 8
10.)	48	18.)	a. 10		e. 23	34.)	72 = 2 x 9 x B
11.)	70		b. 12	23.)	a. 875		
12.)	95		c. 7		b. 25		
13.)	48		d. 8		c. 84,000		
14.)	a. 25		e. 9		d. 360		
	b. 95	19.)	a. 7		e. 2,105		
	c. 70		b.35	24.)	a. 12,106		
	d. 69		c. 108		b. 1,028		
	e. 30		d. 191		c. 564		
15.)	a. 35	20.)	a. 28		d. 8,722		
	b. 30		b. 6,720		e. 6		
	c. 12		c. 67,200				
	d. 34		d. 1,053				
	e. 65		e. 130				

Section 2

Fractions

FRACTIONS

Chapter 3

Naming Fractions

Equivalent Fractions

Reducing Fractions

Lowest Common Denominator

Improper Fractions and Mixed Numbers

Addition or Subtraction of Fractions

Multiplication of Fractions

Division by Fractions

Fractions

- From Latin word *fractus* meaning "broken"
- A *fraction* is a ratio of two represents a part of a whole.

- *Numerator* - "numbering part"
  How many parts are being considered

- *Fraction Bar* – "division line"

- *Denominator* – "names the fraction"
  How many parts the whole was divided into

Fractions - Denominator

- Bottom of fraction
- Indicates total number of equal parts of the whole
- Names the fraction

5 equal pieces of the whole (Fifths)
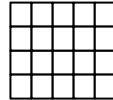
20 equal pieces of the whole (twentieths)
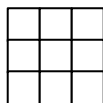
9 equal pieces of the whole (ninths)
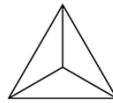
3 equal pieces of the whole (thirds)

8 equal pieces of the whole (eighths)
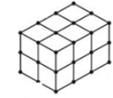
12 equal pieces of the whole (twelfths)

Fractions - Numerator

- Top of fraction
- Indicates number of equal parts of interest

3 parts of the whole

9 parts of the whole
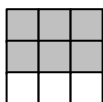
6 parts of the whole

2 parts of the whole

4 parts of the whole
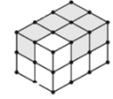
5 parts of the whole

Fractions
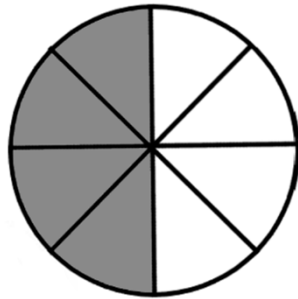
There are 8 wedges in this circle.
We are wanting to color 4 wedges.
How do we represent this as a fraction?

We want 4 of those parts.
(numerator) → $\boxed{4}$

The circle has 8 equal parts. → $\boxed{8}$
(denominator)

$$\frac{4}{8}$$

Fractions

- Indicated the highlighted quantities using fractions.
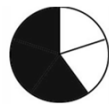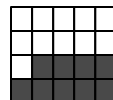
3 pieces of interest	$\dfrac{3}{5}$	9 pieces of interest $\dfrac{9}{20}$
5 total pieces		20 total pieces
6 pieces of interest	$\dfrac{6}{9}$	2 pieces of interest $\dfrac{2}{3}$
9 total pieces		3 total pieces
4 pieces of interest	$\dfrac{4}{8}$	5 pieces of interest $\dfrac{5}{12}$
8 total pieces		12 total pieces
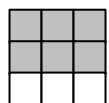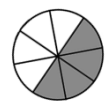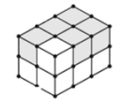
Equivalent Fractions

- *Equivalent fractions* are fractions with different numbers representing the same part of a whole.
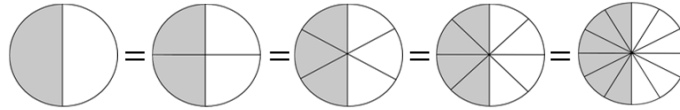
$$\frac{1}{2} \;=\; \frac{2}{4} \;=\; \frac{3}{6} \;=\; \frac{4}{8} \;=\; \frac{6}{12}$$

Finding Equivalent Fractions

- To find an equivalent fraction, multiply or divide the numerator and denominator by the same number.
- Example: Find an equivalent fraction of $4/10$.

$$\frac{4 \; x \; 3 \; = \; 12}{10 \; x \; 3 \; = \; 30} \qquad \frac{4 \; \div \; 2 \; = \; 2}{10 \; \div \; 2 \; = \; 5}$$

$$\frac{4 \; x \; 12 \; = \; 48}{10 \; x \; 12 \; = \; 120}$$

All the products and quotients are equivalent fractions of $\dfrac{4}{10}$.

$$\frac{4 \; x \; 100 \; = \; 400}{10 \; x \; 100 \; = \; 1000}$$

Equivalent Fractions

- To determine if two fractions are equivalent, cross-multiply the fractions.

- Example: Are 1/8 and 5/32 equivalent fractions?
  - Step 1. Rewrite fractions in vertical format and set equal.

$$\frac{1}{8} \diagdown\!\!\!\!\diagup \frac{5}{32}$$

  - Step 2. Cross-multiply numerators and denominators.
$$(1)(32) = (8)(5)$$

  - Step 3. Complete math and verify equation balance.
$$32 \neq 40$$

      The above is not true, so the fractions are not equivalent.

Equivalent Fractions

- Are 1/8 and 5/32 equivalent? (continued)

$$\frac{1}{8} \diagdown\!\!\!\!\diagup \frac{5}{32}$$

  - Step 2. Cross-multiply numerators and denominators.
$$(1)(32) = (8)(5)$$

  - Step 3. Complete math and verify equation balance.
$$32 \neq 40$$

      The above is not true, so the fractions are not equivalent.

Reducing Fractions

- To *reduce* or *simplify* a fraction, divide the numerator and denominator by the same number.
  - Continue until smallest equivalent fraction is achieved.

- Example: Reduce the fraction 8/30 to simplest terms.

$$\frac{8}{30} \div \frac{2}{2} = \frac{4}{15}$$

- Example: Reduce the fraction 45/75 to simplest terms.

$$\frac{45}{75} \div \frac{5}{5} = \frac{9}{15}$$

$$\frac{9}{15} \div \frac{3}{3} = \frac{3}{5}$$

Lowest Common Denominator

- The *lowest common denominator* is the smallest number by which all denominators can evenly be divided.

- Multiplication Method
  - Ask "what is the smallest number that all the denominators can divide into evenly?"
    - *This can be found by multiplying the denominators together.*

- Factoring Method
  - Factor each denominator to lowest terms.
  - List factors and determine greatest number of times each factor occurs.
  - Multiply those factors together to obtain LCD.

- Once, LCD is found, convert all fractions to have the LCD in denominator.

Lowest Common Denominator

- Multiplication Method
  - Multiply denominators together to find a common denominator
    - May not be lowest common denominator
  - Works best with small denominator values
  - Example: Find the LCD for 1/3 and 1/4.

$$\frac{1}{3} \qquad\qquad \frac{1}{4}$$
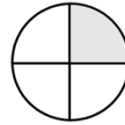
Lowest Common Denominator

- Example: Find the LCD for $1/3$ and $1/4$. (continued)
  - Multiply 3×4 to get "first guess."
$$3 \times 4 = 12$$

- Convert both fractions to denominator of 12.

$$\frac{1}{3} \times \frac{4}{4} = \frac{4}{12} \qquad\qquad \frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$$
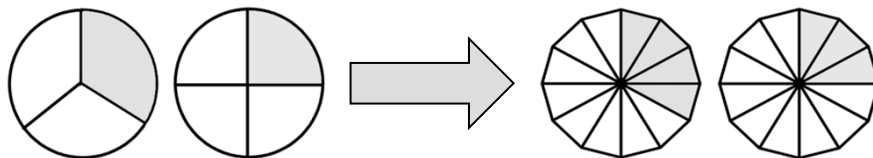
Lowest Common Denominator

- Factoring Method
  1. Factor each denominator to lowest terms (use "factor tree").
  2. List the factors represented in the tree and determine the greatest number of times each factor occurs.
  3. Multiply those factors together to obtain LCD.
  - Example: Find the LCD for 7/16 and 11/12.
  1. Factor each denominator, using the factor tree
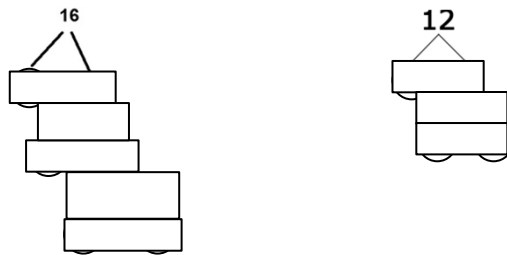
Lowest Common Denominator

- Example: Find the LCD for 11/12. (continued)
  2. List all factors on last (lowest) level of factor tree.

Factor Shown	Greatest # times it occurs
2	(2)(2)(2)(2)
3	(3)

- 2 occurs the <u>greatest</u> number of times (four) in 16 tree
  - (2)(2)(2)(2)
- 3 occurs the <u>greatest</u> number of times (one) in the 12 tree
  - (3)
  3. Multiply factors in "greatest times" list.

$$LCD = (2)(2)(2)(2)(3) = 48$$

Improper Fractions & Mixed Numbers

- A *mixed number* is a whole number plus a fractional part.

$$4\frac{1}{5}$$

- A *proper fraction* has a numerator smaller than the denominator.
- An *improper fraction* is a fraction where the numerator is larger than the denominator.

$$\frac{75}{13}$$

- You can convert between mixed numbers and improper fractions without changing the value of the figure.

Improper Fractions & Mixed Numbers

- To convert a mixed number to an improper fraction:
  1. Multiply denominator by the whole number.
  2. Add product to numerator to determine final numerator.
  3. Reduce fraction to simplest terms.

- Example: Express $2\frac{3}{4}$ as an improper fraction.

  1. Multiply denominator by the whole number
$$(4)(2) = 8$$
  2. Add product to numerator to determine final numerator
$$8 + 3 = 11$$

$$2\frac{3}{4} = \frac{11}{4}$$

  3. Reduce fraction.

Improper Fractions & Mixed Numbers

- Example (cont'd):   Express $2\frac{3}{4}$ as an improper fraction.

$$1 \qquad 1 \qquad \frac{3}{4}$$

$$= 2\frac{3}{4}$$

$$\frac{4}{4} \qquad \frac{4}{4} \qquad \frac{3}{4}$$

$$= \frac{4+4+3}{4} = \frac{11}{4}$$
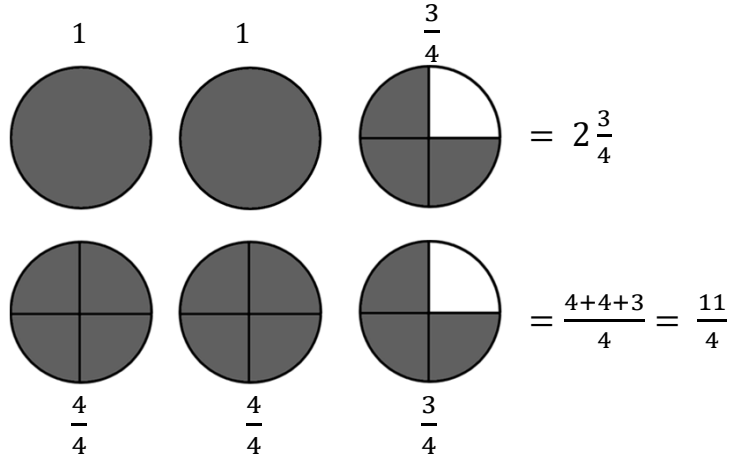
Improper Fractions & Mixed Numbers

- To convert from an improper fraction to a mixed number:
  1. Reduce improper fraction to simplest terms, if possible.

  $$\frac{125}{55} \div \frac{5}{5} = \frac{25}{11}$$

  2. Divide numerator by denominator to obtain whole number.

  $$25 \div 11 = 2.7$$
  $$2 \times 11 = 22$$

  3. Any remainder becomes numerator.

  $$25 - 22 = 3$$
  $$2\frac{3}{11}$$

  4. Reduce fractional part if possible.

  $$2\frac{3}{11}$$

Adding & Subtracting Fractions

- When adding or subtracting fractions, the denominators must be the same.
  - If denominators are not the same, convert fractions to Lowest Common Denominator and perform calculation
- Example (like fractions):   Find the sum of $1/6$ and $4/6$.
-

$$\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$$
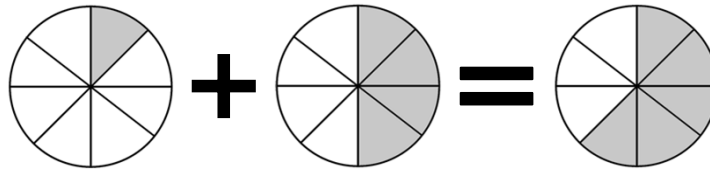
Adding & Subtracting Fractions

- Example: Find the difference between $1/3$ and $1/4$.

$$\frac{1}{3} - \frac{1}{4} = ?$$

- Different sizes cannot be added or subtracted.
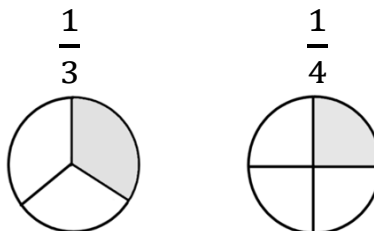
$$\frac{1}{3} \qquad\qquad \frac{1}{4}$$

- Lowest Common Denominator (LCD) must be found.

Adding & Subtracting Fractions

- Example(continued): $\dfrac{1}{3} - \dfrac{1}{4} = ?$

- Find LCD.

$$3 \times 4 = 12$$

- Convert both fractions to common denominator of 12.

$$\dfrac{1}{3} \times \dfrac{4}{4} = \dfrac{4}{12} \qquad\qquad \dfrac{1}{4} \times \dfrac{3}{3} = \dfrac{3}{12}$$
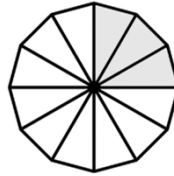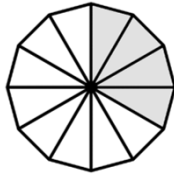
Adding & Subtracting Fractions

- Example (continued): $\dfrac{1}{3} - \dfrac{1}{4} = ?$

$$\dfrac{1}{3} = \dfrac{4}{12} \qquad\qquad \dfrac{1}{4} = \dfrac{3}{12}$$

- Perform calculations & reduce fraction.

$$\dfrac{4}{12} - \dfrac{3}{12} = \dfrac{1}{12}$$

- Denominator does not change as it is indicating the total number of pieces.
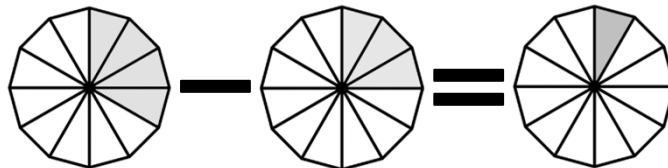
Multiplying Fractions

- When multiplying fractions:
  - Multiply the numerators
  - Multiply the denominators
  - Reduce fraction to lowest terms

- Example: Find the product of 7/10 and 5/8.

$$\frac{7}{10} \times \frac{5}{8} = \frac{7 \times 5}{10 \times 8} = \frac{35}{80}$$

$$\frac{35}{80} \div \frac{5}{5} = \frac{7}{16}$$

Dividing Fractions

- To divide a fraction:
  - Invert (flip) the divisor (denominator).
  - Multiply the fractions.
- Example: Find the quotient of $3/5$ divided by $1/4$.

$$\frac{3}{5} \div \left(\frac{1}{4}\right) = ?$$ OR $$\frac{\frac{3}{5}}{\frac{1}{4}} = ?$$

- Invert (flip) the denominator.

$$\frac{1}{4} \implies \frac{4}{1}$$

- Multiply fractions.

$$\frac{3}{5} \times \frac{4}{1} = \frac{3 \times 4}{5 \times 1} = \frac{12}{5}$$

Combined Calculations w/Fractions

- A complex fraction is a fraction whose numerator and/or denominator contains a fraction.

- Example:

$$\frac{\frac{2}{3}}{3}$$ numerator / division line / denominator

$$\frac{25}{\frac{6}{7}}$$ numerator / division line / denominator

$$\frac{\frac{2}{5}}{\frac{15}{16}}$$ numerator / division line / denominator

---

Combined Calculations w/Fractions

- To complete these problems:
  - Simplify the numerator and denominator.
  - Restate the original problem.
  - Divide as indicated by the restated problem.

- Example: Rewrite the following as a decimal: $\frac{\frac{2}{5}}{\frac{15}{16}}$

$$\frac{(2 \div 5)}{(15 \div 16)} = \frac{0.4}{0.9375} = 0.4 \div 0.9375 = 0.4267$$

- Example: Simplify the following fraction: $\frac{\frac{2}{5}}{\frac{15}{16}}$

$$\frac{2}{5} \times \frac{16}{15} = \frac{(2 \times 16)}{(5 \times 15)} = \frac{32}{75}$$

$$32 \div 75 = 0.4267$$

Math Fundamentals
Fractions Practice Problems

Determine the name of the fraction indicated by the colored portions of the diagrams below.

1. _____

6. _____

2. _____

7. _____

3. _____

8. _____

4. _____

9. _____

5. _____

10. _____
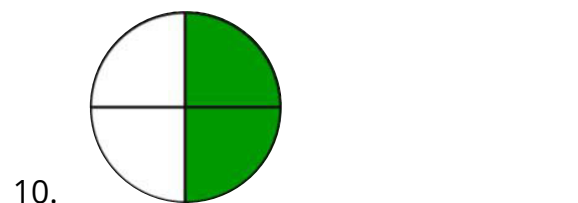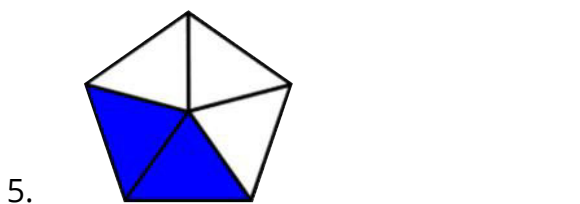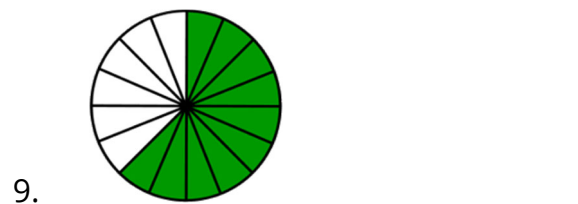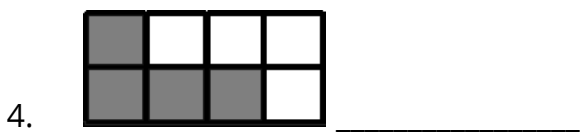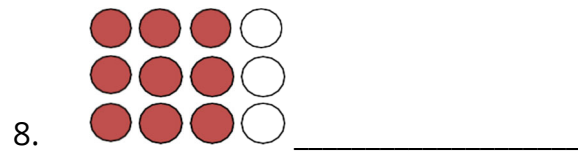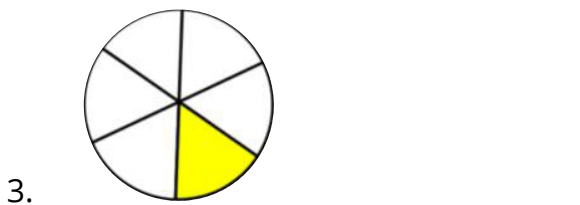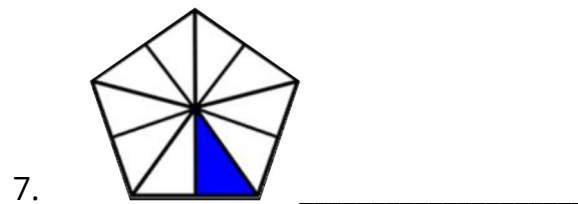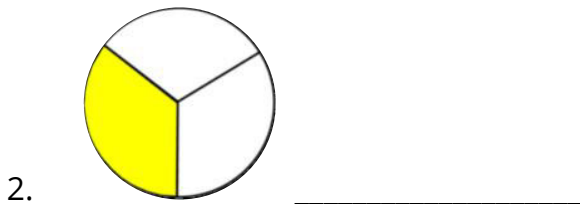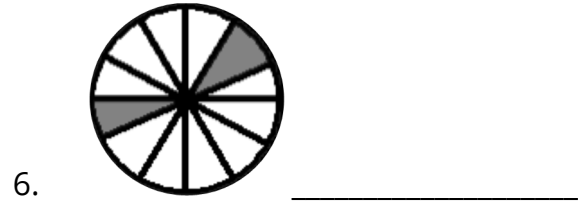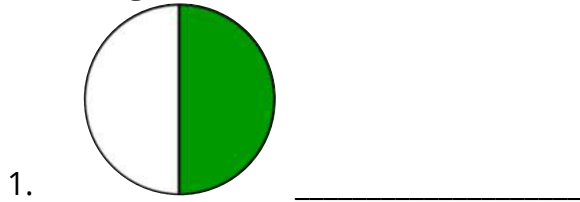
Determine if the following fractions are **equivalent**.

11. $\dfrac{1}{15} = \dfrac{4}{60}$ Yes / No

12. $\dfrac{145}{175} = \dfrac{29}{45}$ Yes / No

13. $\dfrac{60}{66} = \dfrac{10}{13}$ Yes / No

14. $\dfrac{140}{180} = \dfrac{32}{36}$ Yes / No

15. $\dfrac{5}{24} = \dfrac{25}{110}$ Yes / No

16. $\dfrac{28}{72} = \dfrac{7}{18}$ Yes / No

17. $\dfrac{44}{116} = \dfrac{11}{30}$ Yes / No

18. $\dfrac{12}{81} = \dfrac{4}{27}$ Yes / No

19. $\dfrac{115}{160} = \dfrac{32}{23}$ Yes / No

20. $\dfrac{4}{128} = \dfrac{24}{32}$ Yes / No

Reduce the following fractions to their simplest terms. It may be an improper fraction.

21. $\dfrac{45}{90} =$

22. $\dfrac{45}{81} =$

23. $\dfrac{14}{35} =$

24. $\dfrac{37}{103} =$

25. $\dfrac{5}{10} =$

26. $\dfrac{30}{50} =$

27. $\dfrac{56}{20} =$

28. $\dfrac{525}{84} =$

29. $\dfrac{1115}{125} =$

30. $\dfrac{6144}{800} =$

Find the **<u>product</u>** of the following expressions. Provide the answers as a fraction in simplest terms. It may be an improper fraction.

31. $\dfrac{8}{9} \times \dfrac{1}{2} =$ _____

32. $\dfrac{3}{15} \times \dfrac{3}{7} =$ _____

33. $\dfrac{8}{5} \times \dfrac{10}{9} \times \dfrac{26}{7} =$ _____

34. $6 \times \dfrac{9}{20} =$ _____

35. $\dfrac{19}{9} \times \dfrac{11}{10} =$ _____

36. $\dfrac{175}{205} \times \dfrac{3}{4} =$ _____

37. $\dfrac{865}{1649} \times \dfrac{15}{70} =$ _____

38. $20 \times \dfrac{9}{30} \times \dfrac{4}{14} =$ _____

Find the **sum** of the following expressions. Provide the answers as a fraction in simplest terms. It may be an improper fraction.

39. $\dfrac{9}{10} + \dfrac{7}{10} = $ _____

40. $\dfrac{8}{11} + \dfrac{2}{11} = $ _____

41. $\dfrac{125}{1157} + \dfrac{235}{1157} = $ _____

42. $\dfrac{1}{3} + \dfrac{2}{5} = $ _____

43. $\dfrac{3}{6} + \dfrac{1}{2} = $ _____

44. $\dfrac{3}{10} + \dfrac{1}{3} = $ _____

45. $\dfrac{33}{50} + \dfrac{15}{25} = $ _____

46. $\dfrac{23}{25} + \dfrac{9}{30} = $ _____

47. $2 + \dfrac{5}{9} = $ _____

48. $\dfrac{2}{4} + \dfrac{17}{18} = $ _____

Find the **difference** of the following expressions. Provide the answers as a fraction in simplest terms. It may be an improper fraction.

49. $\dfrac{10}{12} - \dfrac{3}{12} =$ _____

50. $\dfrac{6}{10} - \dfrac{5}{10} =$ _____

51. $7 - \dfrac{5}{6} =$ _____

52. $\dfrac{14}{9} - \dfrac{5}{9} =$ _____

53. $\dfrac{28}{12} - \dfrac{11}{12} =$ _____

54. $\dfrac{2}{3} - \dfrac{2}{5} =$ _____

55. $\dfrac{6}{7} - \dfrac{2}{3} =$ _____

56. $\dfrac{15}{16} - \dfrac{4}{6} =$ _____

57. $\dfrac{22}{25} - \dfrac{1}{5} =$ _____

58. $\dfrac{187}{1250} - \dfrac{4}{75} =$ _____

Find the **quotient** for the following expressions. Provide the answers as a fraction in simplest terms. It may be an improper fraction.

59. $\dfrac{1}{5} \div \dfrac{2}{3} =$ _____

60. $\dfrac{3}{7} \div \dfrac{5}{9} =$ _____

61. $\dfrac{4}{6} \div \dfrac{3}{4} =$ _____

62. $\dfrac{1}{9} \div \dfrac{11}{12} =$ _____

63. $\dfrac{\frac{1}{2}}{\frac{1}{12}} =$ _____

64. $\dfrac{\frac{11}{5}}{\frac{20}{3}} =$ _____

65. $\dfrac{2}{5} \div 7 =$ _____

66. $9 \div \dfrac{2}{5} =$ _____

67. $\dfrac{8}{\frac{2}{3}} =$ _____

68. $\dfrac{\frac{19}{27}}{4} = $ _____

Fractions Answers

1.)	$\dfrac{1}{2}$	18.)	Yes	35.)	$\dfrac{209}{90}$	52.)	1
2.)	$\dfrac{1}{3}$	19.)	No	36.)	$\dfrac{105}{204}$	53.)	$\dfrac{17}{12}$
3.)	$\dfrac{1}{6}$	20.)	No	37.)	$\dfrac{2595}{23086}$	54.)	$\dfrac{4}{15}$
4.)	$\dfrac{4}{8}$	21.)	$\dfrac{1}{2}$	38.)	$\dfrac{12}{7}$	55.)	$\dfrac{4}{21}$
5.)	$\dfrac{2}{5}$	22.)	$\dfrac{5}{9}$	39.)	$\dfrac{8}{5}$	56.)	$\dfrac{13}{48}$
6.)	$\dfrac{2}{12}$	23.)	$\dfrac{2}{5}$	40.)	$\dfrac{10}{11}$	57.)	$\dfrac{17}{25}$
7.)	$\dfrac{1}{10}$	24.)	$\dfrac{37}{103}$	41.)	$\dfrac{360}{1157}$	58.)	$\dfrac{361}{3750}$
8.)	$\dfrac{9}{12}$	25.)	$\dfrac{1}{2}$	42.)	$\dfrac{11}{15}$	59.)	$\dfrac{3}{10}$
9.)	$\dfrac{10}{16}$	26.)	$\dfrac{3}{5}$	43.)	1	60.)	$\dfrac{27}{35}$
10.)	$\dfrac{2}{4}$	27.)	$\dfrac{14}{5}$	44.)	$\dfrac{19}{30}$	61.)	$\dfrac{8}{9}$
11.)	Yes	28.)	$\dfrac{25}{4}$	45.)	$\dfrac{63}{50}$	62.)	$\dfrac{4}{33}$
12.)	No	29.)	$\dfrac{223}{25}$	46.)	$\dfrac{183}{145}$	63.)	6
13.)	No	30.)	$\dfrac{192}{25}$	47.)	$\dfrac{14}{9}$	64.)	$\dfrac{33}{100}$
14.)	No	31.)	$\dfrac{4}{9}$	48.)	$\dfrac{13}{9}$	65.)	$\dfrac{2}{35}$
15.)	Yes	32.)	$\dfrac{3}{35}$	49.)	$\dfrac{7}{12}$	66.)	$\dfrac{45}{2}$
16.)	Yes	33.)	$\dfrac{416}{63}$	50.)	$\dfrac{1}{10}$	67.)	12
17.)	No	34.)	$\dfrac{27}{10}$	51.)	$\dfrac{7}{6}$	68.)	$\dfrac{19}{108}$

Section 3

Decimals & Percentages

DECIMALS

Chapter 4

The Decimal System
Addition and Subtraction of Decimals
Multiplication of Decimals
Division of Decimals
Converting Decimals and Fractions

Decimal System

- The word decimal comes from the Latin word meaning *decem,* meaning ten.

- The decimal system is based on ten and multiples of ten.

- In a place value system, the size of any number depends on two things:
  - Which digits are used, and
  - Where these digits are placed in relation to the decimal point.

thousands	hundreds	tens	ones	Decimal Point	tenths	hundredths	thousandths	Ten thousandths
1000	100	10	I	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10,000}$

Greater than one ← — — — — — — — — — — → Less than one

Place Value System

- Value of any number depends on two things:
  1. Which digits are used (0, 1, 2…9)
  2. Where these digits are placed in relation to the decimal point

- Example: 1725

  172.5   1.725   0.01725   1725.0

- *Whole numbers* are any of those numbers to the left of the decimal point.

- *Decimal fractions* are the number to the right of the decimal and represent quantities less than 1.
  - Same as a fraction shows parts of the whole

Decimal Numbers

- Example: Express the shaded area of the following as a mixed number and a decimal number.

  1   +   1   +   7/10
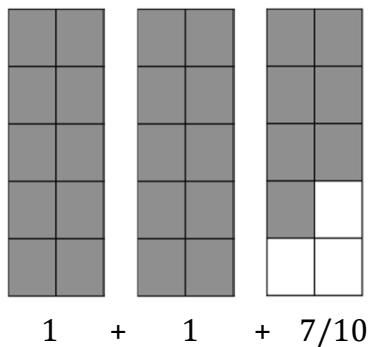
  - Mixed Number:
    $$2\frac{7}{10}$$
  - Decimal Number:
    2.7
  - Read as:
    Two and seven tenths

Adding & Subtracting Decimal Numbers

- When adding or subtracting decimal numbers:
  - Line up decimal points

- Example: Find the sum of 3.41 and 0.125.

$$
\begin{array}{r}
3.41 \\
+0.125 \\
\hline
3.536
\end{array}
$$

- Example: Find difference between 275.14 and 78.2.

$$
\begin{array}{r}
275.14 \\
-78.2 \\
\hline
196.94
\end{array}
$$

Multiplying Decimal Numbers

- When multiplying decimal numbers:
  - Multiply the numbers, disregarding the decimal points.
  - Using all factors, add up the total number of places to the right of the decimal point.
  - Place the decimal point in answer.

- Example: Find the product of 1.7 and 23.89.

$$17 \times 2389 = 40613$$

$$1.7 \qquad 23.89$$

Total of **3** integers to the right of the decimal.

$$\boxed{40.613}$$

Dividing Decimals

- To divide decimal numbers by whole numbers:
  - Place the decimal point in the answer.
  - Divide as with whole number.
  - Example: Divide 145.7 by 3.

- To divide by decimal numbers by whole numbers:
  - Move decimal point in the divisor to far right.
  - Move decimal point in dividend the same number of places to the right.
  - Place the decimal in the answer.
  - Divide as with whole numbers.
  - Example: Complete the following 20.9 ÷ 0.55.

Converting Decimals & Fractions

- To convert a decimal to a fraction:
  - Disregarding the decimal, place entire number in numerator.
  - Place value of the last number to the right indicates the denominator of the fraction.
  - Example: Express 0.53 as a fraction.

$$0.53 = \frac{53}{100}$$

- To convert a fraction to a decimal:
  - Write the fraction as a division problem and solve.
  - Example: Express 4/7 as a decimal.

$$\frac{4}{7} = 4 \div 7 = 0.5714$$

Math Fundamentals
Decimal & Percent Practice Problems

Convert the following decimals to fractions.

1.) 1.95 =   3.) 0.769 =   5.) 1.89 =

2.) 0.923 =   4.) 1.74 =   6.) 0.5 =

Convert the following fractions to decimals.

7.) $\dfrac{31}{40}$ =   9.) $\dfrac{5}{20}$ =   11.) $\dfrac{475}{50}$ =

8.) $\dfrac{11}{20}$ =   10.) $\dfrac{12}{10}$ =   12.) $\dfrac{17}{40}$ =

Convert each fraction to hundredths (100 in denominator). Then to a convert to a decimal number.

13.) $\dfrac{1}{4}$ =   18.) $\dfrac{37}{50}$ =

14.) $\dfrac{1}{5}$ =   19.) $\dfrac{18}{25}$ =

15.) $\dfrac{4}{20}$ =   20.) $\dfrac{42}{50}$ =

16.) $\dfrac{16}{20}$ =   21.) $\dfrac{1}{2}$ =

17.) $\dfrac{11}{20}$ =   22.) $\dfrac{2}{5}$ =

Convert the following decimals to percent.

23.) 0.44 =

25.) 0.03 =

27.) 1.61 =

24.) 3.82 =

26.) 0.69 =

28.) 0.744 =

Convert the following percents to decimals.

29.) 17.9% =

31.) 13.0% =

33.) 0.8% =

30.) 15.4% =

32.) 7.5% =

34.) 190% =

Convert the following fractions to percents.

35.) $\dfrac{39}{20}$ =

37.) $\dfrac{2}{25}$ =

39.) $\dfrac{12}{10}$ =

36.) $\dfrac{93}{50}$ =

38.) $\dfrac{13}{20}$ =

40.) $\dfrac{5}{25}$ =

Convert the following percents to fractions. Give answer in reduced form.

41.) 83% =

43.) 88.5% =

45.) 175% =

42.) 25% =

44.) 66.5% =

46.) 0.91% =

Solve the following problems.

47.)   What percent of 972 is 291.6?   54.)   What is 5% of 615?

48.)   What percent of 840 is 546?   55.)   What percent of 17 is 0.85?

49.)   What percent of 889 is 195.58?   56.)   What percent of 923 is 692.25?

50.)   What percent of 414 is 347.76?   57.)   What percent of 400 is 100?

51.)   What is 50% of 796?   58.)   What is 15% of 318?

52.)   What is 35% of 560?   59.)   What is 15% of 938?

53.)   What is 10% of 790?   60.)   What is 25% of 54?

Word Problems

Write an expression to represent each question. Then solve the expression.

61.)   There are 4.5 liters of milk in a pot. The chef adds 2.5 g of salt and 1.2 liters of water. How much liquid is in the pot?

62.)   The chef took out 54.8 g of butter but then found out that he took out 6.9 g of butter more than he need. How much butter did he need?

63.)   The chef is making 10.5 batches of dinner rolls. For each batch, he kneaded dough for 0.7 hours and let the dough rest for 1.6 hours. Then he baked the dough for 0.4 hours. How long does it take to make each batch of dinner rolls?

64.)   There are 19.3 cups of flour in a bag. The chef uses 2.9 cups for making cookies. Then, he uses 1.2 cups of sugar and 5.1 cups of flour for making banana bread. How many cups of flour are left?

65.)   There are 25 students in a class and 13 of these students passed their English test. What percentage of these students passed their test? Round your answer to the nearest hundredth if necessary.

66.)   For one Biology test, Sally had to answer 43 questions. Of these 43 questions, Sally answered 31 of them correctly. What percent did Sally get correct on her biology test?

67.)   48 out of 60 hockey players on a team have mastered skating backwards. Write a decimal that represents the players on the team that have not mastered skating backwards.

68.)   20 out of 25 students in a school learn horseback riding. Write a decimal that represents the part of the students that learns horse riding?

69.)   Nancy went to her local zoo where 71.43% of its exhibits featured bears. If the zoo features 42 exhibits in total, how many of the zoo's exhibits feature bears?

70.)   In one particular suburb, 29.41% of families own a Labrador. If there are a total of 17 families in this neighborhood that own a dog in general, then how many dog owners own a Labrador?

71.)   Carlo cut an excess of 8.7 inches off a 98.5-inch-long iron rod for the reconstruction of the gate at his house. He cut it again into 2 pieces of the same length which he needs for both sides of the gate. How long was the iron rod after cutting the excess? How long was the iron rod needed for each side of the gate?

72.)   One baseball team played 20 games throughout their entire season. If this baseball team won 8 of those games, then what percentage of their games did they win?

73.)   For one History test, Peter correctly answered 21 questions. These correct answers gave him a percent score of 63.64%. In total, how many questions were on this History test?

74.)   Tim is a postman that works at a local post office. The basic postage is $1.25 and covers any letter that is lighter than 7.5 grams. For heavier letters, extra postage of $0.87 is needed for every 10 grams over 7.5 grams.

    a. Can a letter that is 705/100 grams be covered by the basic postage?

    b. Emma mailed a letter that weighs 12.92 grams. Compared to the weight covered by basic postage, how much heavier is Emma's letter?

    c. What is the postage needed for a letter that is 37.5 gram?

    d. On average, the post office processes 45.6 letters every hour. How many letters can the office process during the 9 hours it is open?

    e. During Christmas, the post office opens 12 hours a day. Compared to usual days, how many more letters can the post office process?

75.)   Tim works on a 6-hour shift today. He has 2 bags of mail that he needs to deliver today. One is 24.52 kilograms and the other one is 18.8 kilograms. What is the total weight of the two bags of mail?

76.)   The old sewing machine is 8 inches long and 17.3 inches wide while the brand-new sewing machine is 13.7 inches wide. How much wider is the old sewing machine than new sewing machine?

77.)   Farmer Alex lives 0.35 km away from the farm. His co-worker Jordan lives 39/100 km away from the same farm. Whose house is farther from the farm? By what distance?

78.)   Benny receives a $18000 salary for working as an executive. If Benny spends 33.33% of his salary on expenses each year, then how much money does Benny have to spend on expenses?

79.)   If among 40 classroom students, 16 go to band practice, write a decimal that represents the number of students from the class that are not in band?

80.)   A seamstress bought 3 types of fabric. The cotton fabric is 17.2 yards long. The silk fabric is 7.8 yards long and the wool fabric is 2.3 yards longer than the silk fabric. What is the total length of fabric that the seamstress bought?

81.)   One customer came and wanted to adjust her long skirt up to her knee. Her skirt was 76.8 inches long. The seamstress measured her waist up to her knee which was 56.5 inches long. What length does the seamstress need to adjust in the skirt?

82.)   A seamstress had 48.2 meters of white sewing thread and 12.3 meters of yarn. She used 9.2 meters of thread for the pants and another 5.3 meters of thread for dress. How long of sewing thread left?

83.)   The chef filled a big pot with 3.7 cups of water and filled a smaller pot with
        7.3 cups of water. How much more water is in the small pot?

84.)   In his garage, Arthur found a piece of wood with defective ends which was
        427.12 cm long. He cut 5.32 cm off one end and another 3.75 cm off the other
        end. How long was the piece of wood after cutting its ends?

85.)   The company ordered boxes of concrete nails weighing 3.35 lbs each and a
        box of 2.5 lbs of screws. The eight construction workers each received a box of
        nails and each worker used 1.23 lbs of concrete nails yesterday. What is the
        weight of the concrete nails left in each box? If the concrete nails were put
        together in a big toolbox, how much would the remaining concrete nails weigh?

86.)   Ashley is making cookies for her office's Christmas party.
        a. Each batch of cookie mix need 0.4 cups of sugar, and each batch can
            make 16 cookies. If Ashley is making 4 batches of cookies, how much
            sugar does she need?

        b. Ashley measured 1.43 cups of sugar. How much more sugar does she
            need?

        c. She has 3 bags of flour. She has two smaller bags with 0.75 kg of flour
            each and a bigger bag which has 1.14 kg. How much flour does she have
            in total?

        d. After she finished making the cookies, Ashley had 0.945 kg of flour left.
            How much flour did she use from the 3 bags?

e. Each batch of cookies is 8.9 oz. What is the weight of 2.5 batches of cookies?

87.) There are two containers of butter. The blue container has 10.6 oz butter, and the green container has 9.9 oz butter. How much butter is there in total?

88.) On a farm, the farmer harvested 30.7 crates of apples and 27.9 crates of oranges. How much more crates of apples are harvested than oranges?

89.) The birch tree outside the farm is 3.5 meters tall. The pine tree is 5.4 meters tall. The maple tree is 1.1 meter taller than the birch tree. Which is taller, the maple tree or the pine tree? How much taller is the pine than the birch?

90.) Farmer Joe uses 9.5 bags of fertilizer for the pumpkin field. He uses 0.7 more bags of fertilizers for the tomato field. Compared to the tomato field, he uses 1.3 less bags of fertilizers for the cabbage field. How many bags of fertilizers does he uses for the cabbage field?

91.) A seamstress was selling her handmade flowering dress for $8.30 and a sling bag for $9.80. Her friend wants to buy that dress and requests for a discount. Seamstress gave her a discount of $1.70. How much does her friend need to pay for the dress?

92.)   95 out of 150 students in a school learn to computer program. Write a decimal that represents the number of students that did not learn to computer program?

93.)   At a construction job for a gallery, 3 painters were tasked with painting the interior. These painters made up 20.00% of the painting crew, so how many painters in all worked on this job?

94.)   Nancy decided to look at new and used vans. Nancy found a used van for $8000. Nancy found that she paid 40% of the price of a new van. How much would a new van cost?

95.)   The owner of the farm has 5.6 acres of land planted with banana trees, 4.35 acres of land for tomato plants and the rest is still unoccupied. If he owns a total of 12.56 acres of land, how large is the unoccupied piece of land does he has?

96.)   Johnny was paid $5.60 for a job he did on the farm for 1 hour. At the same rate, how much would he be paid if he did the same job for 3.5 hours?

97.)   At a local department store, skirts typically cost $31. However, due to a special, the skirts are reduced to $19. What percent of the original price has the skirts been reduced to?

98.)   One baseball team won 4 games throughout their entire season. Of all their games, this team won 25% of them. Given this, how many games in total did this baseball team play?

99.)   The fruit dealer sold 23.67 kgs of mangoes, 67.12 kgs of watermelons, 12.38 kgs of strawberries and 56.73 kgs of oranges. How many kilograms of fruits were left if the fruit dealer started with 167.45 kgs of fruit?

100.)   The first 50 buyers of the album of the acoustic band will receive a discount of $5.36. Gladys was the 46th buyer of the album for which the original price was $37.25. How much would Gladys pay for the album?

101.)   Abby's mobile phone has a storage capacity of 32 gigabytes. She used 15.50 gigabytes for 5 movies and 12 pictures on her phone. Yesterday, she downloaded 2.36 gigabytes of music videos from her favorite band. How much space was still available on her phone?

102.)   Honey earns money by playing online games for points. For every 1,000 points, Honey will receive $3.34. She has earned a total of 8500 points from that online game. How much money will she receive?

103.)   While mining, John found a large metal bar that contained 15 ounces of silver. John also determined that the bar was 55.56% silver. What was the total weight of the metal bar in ounces?

104.) At a customer service center, a short call is defined as any call that is shorter than 5.5 minutes. The average call time on weekdays is 6.4 minutes. The average call time on weekends is 9.38 minutes.

    a. What is the difference between the average call time on weekdays and weekends?

    b. On a particular call, the customer spoke to one representative for 2.4 minutes and then was transferred to a manager for another 2.9 minutes. Was this call a shorter than average call?

    c. If a call received on a Sunday is 12.45 minutes longer than the average time, how long was that call?

    d. On a Monday, each customer service representative received 8.7 calls per hour. What was the average time spent on calls per hour for each staff?

    e. In a 3-hour shift on the same Monday, on average how much time was not spent on calls?

    f. Each customer service representative is paid $12.15 per hour. How much was each representative paid for each 6-hour shift?

105.) Every transaction for online banking made on the bank website costs $1.34. Gerry made 3 transactions while his co-worker Henry made 7 transactions on the online banking website last week. How much money did Henry pay for those transactions last week?

106.) Annie spends an average of 2.77 hours on online gaming and 2.44 hours on doing other school activities online each day, 6 days a week. About how long does she spend online in a day? In a week?

107.) While mining, Peter found a large metal bar that weighed 35 grams. Peter was also able to determine that the bar contained 65.71% silver. How many grams of silver are in the metal bar?

108.) At a local department store, slacks are typically priced at $39. Due to a special, the slacks are reduced to 69.23% of their original price. How much are slacks now?

109.) Keith went to his local zoo that featured 26 canine exhibits. If the zoo features 38 exhibits in total, then what percent of the zoo's exhibits feature canines?

110.) Benny decided to look at new and used trucks. Benny found a used truck for $32,000. A new truck is $44,000, so what percent of the price of a new truck does Benny pay for a used truck?

111.) Sara receives a $21,000 salary for working as an agent. If Sara spends 42.86% of her salary on expenses each year, then how much money does Sara have to spend on expenses?

112.) At a construction job for a mansion, 22 painters were tasked with painting the interior. These painters made up 63.63 % of the painting crew, so how many painters in all worked on this job?
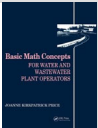
113.) There are 20 students in a class and 40% of these students passed their history test. What number of these students failed their test?

114.) The price for a pumpkin last year was $1.40 per kilogram. The price for a pumpkin this year has increased by $0.80 per kilogram. What is the price for pumpkin this year?

115.) Farmer Joe has 12.4 acres of land. He uses 4.5 acres of land for growing fruits and 6.1 acres of land for growing vegetables. How much land does he have left to use?

116.) The construction worker works for $15.50 per hour, but he does not get paid for his lunch time. He spent 3.8 hours for one task, 1.3 hours for lunch and 4.4 hours on another on-site task yesterday. How much would the construction worker be paid?

117.) Alex can do a certain construction task in 3.5 hours. Roger can do the same task in 2.35 hours. How much faster is Roger than Alex?

118.) Ariel needs 7.84 bags of cement and 14.3 meters of iron rod for the construction of one panel of concrete fence. How many bags of cement does she need for 12 panels of concrete fence?

119.)   At the local fruit market, each regular size basket contains 40 mangoes and costs $61.50. The farmer harvested 20.2 baskets of mangoes yesterday.
    a. How many mangoes were harvested?

    b. How much was the total cost of the baskets the farmer harvested yesterday?

120.)   The farmer bought 2.5 sacks of fertilizer for $2.20 per sack and 4.5 sacks of essential soil for $1.80 for each sack from the supplier. If he gave a $15 bill to the supplier, how much would his change be?

121.)   Cherry called her aunt via an international call with a rate of $1.21 for the first minute of calling and $0.35 for every other minute. If she spent 8.2 minutes chatting with her aunt abroad, how much would Cherry need to pay?

122.)   There is a collection of 8 candy bags. 5 bags have the candy removed from them. Write a decimal that represents the number of bags with candy in them.

123.)   A bag of 50 biscuits has 5 removed from it. Write a decimal to represent the number of biscuits still in the bag?

124.)   Nelly buys 28 chocolates. 17 of the chocolates have caramel in them. Write a decimal that represents the number of chocolates that do not have any caramel in them.

Decimal and Percent Answers (#1-60)

1.)	$^{39}/_{20}$	21.)	0.50	41.)	$^{83}/_{100}$
2.)	$^{923}/_{1000}$	22.)	0.40	42.)	$^{1}/_{4}$
3.)	$^{769}/_{1000}$	23.)	44%	43.)	$^{177}/_{200}$
4.)	$^{87}/_{50}$	24.)	164%	44.)	$^{133}/_{200}$
5.)	$^{189}/_{100}$	25.)	4%	45.)	$^{7}/_{4}$
6.)	$^{1}/_{2}$	26.)	69%	46.)	$^{91}/_{10000}$
7.)	0.775	27.)	165%	47.)	30%
8.)	0.55	28.)	74.4%	48.)	65%
9.)	0.2	29.)	0.179	49.)	22%
10.)	1.2	30.)	0.154	50.)	84%
11.)	0.94	31.)	0.13	51.)	398
12.)	0.425	32.)	0.075	52.)	196
13.)	0.25	33.)	0.008	53.)	79
14.)	0.20	34.)	0.190	54.)	30.75
15.)	0.20	35.)	195%	55.)	14.45
16.)	0.80	36.)	186%	56.)	75%
17.)	0.55	37.)	8%	57.)	25%
18.)	0.74	38.)	65%	58.)	47.7
19.)	0.72	39.)	120%	59.)	140.7
20.)	0.84	40.)	2%	60.)	13.5

Decimal and Percent Answers (#61-124)

61.)	6.7	82.)	42.9	104.)	a. 2.98
62.)	47.9	83.)	3.6		b. yes
63.)	2.7	84.)	418.05		c. 21.83
64.)	11.3	85.)	16.96		d. 55.68
65.)	52%	86.)	a. 1.6		e. 19.2
66.)	72.09%		b. 0.16		f. 72.9
67.)	0.8		c. 2.64	105.)	9.38
68.)	0.8		d. 1.695	106.)	5.21; 31.26
69.)	30		e. 22.25	107.)	23
70.)	5.0	87.)	20.5	108.)	27
71.)	44.9	88.)	2.8	109.)	68.42
72.)	40%	89.)	1.9	110.)	72.72%
73.)	33	90.)	8.9	111.)	9,000.60
74.)	a. Y; 7.0	91.)	6.6	112.)	16
	b. 5.42	92.)	0.367	113.)	12
	c. 3	93.)	15	114.)	2.2
	d. 410.4	94.)	20,000	115.)	1.8
	e. 19.2	95.)	2.61	116.)	256.16
75.)	43.3	96.)	19.6	117.)	1.15
76.)	3.6	97.)	61.29	118.)	94.08
77.)	Jordan; 0.04	98.)	16	119.)	a. 808
78.)	5999.40	99.)	7.55		b. 1,242.30
79.)	0.6	100.)	31.89	120.)	1.40
80.)	35.1	101.)	14.14	121.)	3.73
81.)	20.3	102.)	28.39	122.)	0.625
82.)	42.9	103.)	27	123.)	0.9
				124.)	0.61
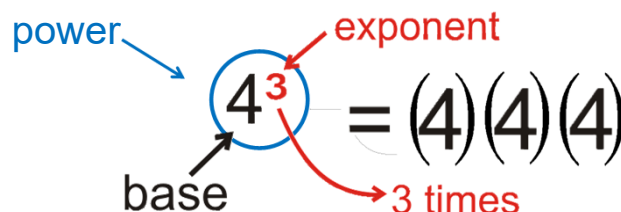
Section 4

Powers, Roots,
& Scientific Notation

Powers, Roots, and Scientific Notation

Chapter 13
Positive Exponents
Negative Exponents
Fractional Exponents
Multiplying and Dividing Powers
Powers of Ten
Scientific Notation

Terminology

- The *base* is the number that is being multiplied.
- An *exponent* refers to the number of times a number is multiplied by itself.
  - Written as a small number to the right and above the base number
  - Also called powers or indices
- The entire expression is called a *power*.

Exponents

- An *exponent* refers to the number of times a number is multiplied by itself.
  - Allows a "shorthand" method of writing multiplication of the same factor two or more times.
- Units and letters act the same as number.
- When a number is written with an exponent, it is said to be in *exponential notation*.

- When all the factors are written out, the number is in *expanded form*.

- Any number or letter that does not have exponent is considered to have an exponent of one.

Positive Exponents

- To write a number in expanded form:
  - Set the base number to multiply by itself the number of times indicated by the exponent
  - Example: Express the following numbers in expanded form.

$$6^2 = ?$$
$$6^2 = (6)(6)$$

$$20^4 = ?$$
$$20^3 = (20)(20)(20)(20)$$

$$\text{ft}^3 = ?$$
$$\text{ft}^3 = (\text{ft})(\text{ft})(\text{ft})$$

Positive Exponents

- To write a number in exponential notation:
  - Count the number of times each number (*base*) is multiplied by itself.
  - The number of times multiplied becomes the exponent.
  - Example: Write the factors shown below using exponential notation.

$$(4)(4)(4) = ?$$
$$? = 4^3$$

$$(12)(12)(12)(12)(12)(12)(12) = ?$$
$$? = 12^7$$

$$(\text{inch})(\text{inch}) = ?$$
$$? = \text{in}^2$$

Negative Exponents

- A factor with a negative exponent can be inverted and written with a positive exponent.
  - When a power is moved from the numerator of a fraction to the denominator, or vice versa, the sign of the exponent must be changed.

$$5^{-2} = \frac{1}{5^2}$$

- Any number that has an exponent of zero (0) is equal to 1.

Roots

- A *root* is a number which, when multiplied together a given number of times, equals the original number.
  - Square root: factor when multiplied together three (2) times
  - Cubed root: factor when multiplied together three (3) times
  - If a number is not written with the root, it is assumed to be 2.

- A *radical* is used to indicate a root is to be taken.

- A *radicand* is the number inside a radical symbol.

Roots

- The *index* indicates the root to be taken.
  - The small number tucked inside the check mark portion of the radical sign.

- Noted one of two ways:
  1. By using a radical:

     $$\sqrt[3]{1728^1}$$

  2. By using a fractional exponent:

     $$1728^{1/3}$$

index $\longrightarrow$ $\sqrt[n]{x^m}$ $\longleftarrow$ radical
$\longleftarrow$ power

radicand

$x^{m/n}$ $\longleftarrow$ power
$\longleftarrow$ index

radicand
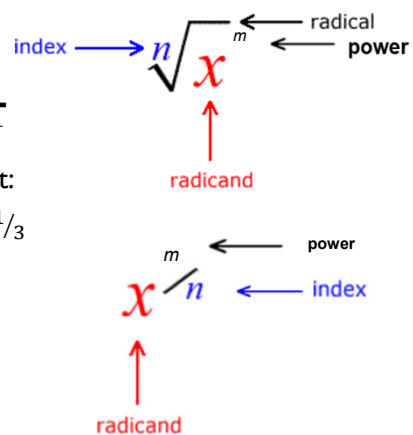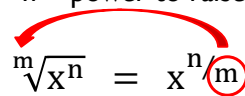
Fractional Exponents

- A *fractional exponent* indicates that a root is to be taken.
  - The denominator determines which root is to be taken.
  - The numerator is the power of the base number.

$$x^{n/m} = \sqrt[m]{x^n}$$

where, m = the root to be taken, and
n = power to raise the base

$$\sqrt[m]{x^n} = x^{n/m}$$

"n" stays with the radicand (as the exponent)
"m" gets moved to the front of the radical (as the index)

Roots & Fraction Exponents

- Example: Express the following numbers using radicals.

$$36^{3/2} = \sqrt[2]{36^3}$$

$$10.4^{5/6} = \sqrt[6]{10.4^5}$$

$$64^{1/3} = \sqrt[3]{64^1} = \sqrt[3]{64}$$

- Example: Express the following numbers using fractional exponents.

$$\sqrt{5^3} = 5^{3/2}$$

$$\sqrt[3]{x^2} = x^{2/3}$$

$$\sqrt[5]{150} = 150^{1/5}$$

Multiplying & Dividing Powers

- When multiplying powers with the same base, simply add the exponents.

$$(x^2)(x^3) = ?$$
$$x^{2+3} = x^5$$
$$(x)(x)(x)(x)(x) = x^5$$

- When dividing powers with the same base, subtract the exponents (numerator minus denominator).

$$\frac{N^7}{N^4} = ?$$

$$\frac{(N)(N)(N)(N)(N)(N)(N)}{(N)(N)(N)(N)} = ?$$

Eliminate common factors in numerator and denominator.

$$(N)(N)(N) = N^3$$

$$N^{7-4} = N^3$$

Powers of Ten (10)

- When multiplying by a power of 10, move the decimal point to the right the same number of places as the power of ten.
  - Example: Rewrite the following expression as a decimal.

$$(3.6)(10^3) = ?$$
$$(3.6)(1000) = 3,600$$
$$3.6 = 3,600$$

- When dividing by the power of 10, move the decimal point to the left the same number of places as the power of ten.
  - Example: Rewrite the following expression as a decimal.

$$\frac{184.4}{10^2} = ?$$

$$\frac{184.4}{100} = 1.844$$

$$184.4 = 1.844$$

Scientific Notation

- *Scientific notation* is a way of writing very large or very small numbers.
  - A number is written in scientific notation when a number between 1 and 10 is multiplied by a power of 10.

- To put a number in scientific notation:
  - Place decimal point after first nonzero digit – *standard position*.
  - Count number of places from standard position to original decimal point – becomes exponent of 10.

Scientific Notation

- To put a number in scientific notation:

    If move decimal point to right, exponent is positive.
    Example: Express 0.00000891 using scientific notation.

$$0.00000891 =$$

    If move decimal point to left, exponent is negative.
    Example: Express 78,882 using scientific notation.

Scientific Notation

- When multiplying or dividing numbers in scientific notation:
    1. Separate the factors of ten and simplify the expression.
    2. Multiply or divide the other factors as indicated by problem.
    3. Leave the answer in scientific notation or take it out as desired.

    Example.

- To take a number out of scientific notation:
    - Multiply by the power of ten as indicated.
        - Positive exponent = move decimal point to right
        - Negative exponent = move decimal point to left

Math Fundamentals
Exponents, Radicals, and Scientific Notation

Write the following exponential expressions in expanded forms.

1. 7^2 = _____

2. 3^4 = _____

3. 5^3 = _____

4. 2^1 = _____

5. x^5 = _____

6. 7^{-2} = _____

7. 43^0 = _____

8. 1.1^3 = _____

9. x^{-3} = _____

10. $9^4 + 3^2$ = _____

Write the following expressions in exponential form.

11. 5 x 5 x 5 x 5 x 5 x 5 = _____

12. 20 x 20 x 20 = _____

13. $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ = _____

14. $51 \times 51 \times 51 \times 51 \times 51 \times 51 \times 51 \times 51 \times 51 \times 51 \times 51 \times 51$ = _____

15. $2.3 \times 2.3 \times 2.3 \times 2.3 \times 2.3 \times 2.3 \times 2.3 \times 2.3 \times 2.3 \times 2.3 \times 2.3 =$ _____

16. $\dfrac{1}{(x)(x)(x)} =$ _____

17. $11 \times 11 \times 11 \times 11 =$ _____

18. $7 =$ _____

19. $\dfrac{3}{7} \times \dfrac{3}{7} \times \dfrac{3}{7} =$ _____

20. $(4 \times 4 \times 4) + (2 \times 2) =$ _____

Write the following exponential expressions in radical form

21. $5^{\frac{2}{3}} =$ _____

22. $6^{\frac{1}{4}} =$ _____

23. $x^{\frac{5}{2}} =$ _____

24. $9^{\frac{3}{4}} =$ _____

25. $144^{\frac{1}{2}} =$ _____

Write the following radicals in exponential form. Exponents should be expressed in fraction form.

26. $\sqrt[6]{3^2} =$ _____

27. $\sqrt[3]{17^4} =$ _____

28. $\sqrt{135^3}$ = _____

29. $\sqrt[4]{x}$ = _____

30. $\sqrt[5]{31^{2.4}}$ = _____

Simply the following expressions. Answers should be written as integers or decimals.

31. $3^3 \times 3^4$ = _____

32. $2^4 \times 2^5$ = _____

33. $6^8 \times 6^0$ = _____

34. $(2^4)^3$ = _____

35. $2^3 \times 2^{-2}$ = _____

36. $4^3 + 8^2$ = _____

37. $(5^4) \times (3^4)$ = _____

38. $(x^{-6}) \times (x^8)$ = _____

39. $17.2^3 \times 17.2^{-4}$ = _____

40. $\frac{1}{9^5} \times 9^5$ = _____

Write the following expressions in scientific notation.

41. $6,050,000,000$ = _____

42. 84,179,000 = _____

43. 0.000000007010 = _____

44. 5,160 = _____

45. 78,882 = _____

46. 0.0000000334040 = _____

47. 0.07035 = _____

48. 25,000,000 = _____

49. 0.65 = _____

50. 9.78 = _____

Write the following expressions in standard notation.

51. 7.93×10^{-1} = _____

52. 7.42×10^{-4} = _____

53. 9.713×10^{5} = _____

54. 1.256×10^{3} = _____

55. 8.3×10^{-6} = _____

56. 4.23×10^{0} = _____

57. 0.0301×10^{7} = _____

58. $2.69 \times 10^{-9} =$ _____

59. $1.151 \times 10^{5} =$ _____

60. $7.37 \times 10^{1} =$ _____

Solve the following word problems. Show answers in scientific notation.

61. The bedroom of our house is 1,200 cubic meters. We know that there are 3.4×10^9 particles of dust per cubic meter. Write how many particles of dust are present in the bedroom of our house.

62. Find out the weight of 6 billion dust particles, if a dust particle has a mass of 7.53×10^{-10} g.

63. Last month, my friend bought a computer. If it can perform 4.66×10^8 calculations per second, what is the performance of the computer in one minute?

64. In Australia, the people use approximately 2,240,000,000 pounds of bread in a year. How can we write this number in scientific notation?

65. If a satellite travels 62,000,000 miles from Earth, how can we write it in scientific notation?

66. The wavelength of yellow light is 0.000065. Can you express this measurement using scientific notation?

67. If the speed of light is 3×10^8 meters/second, how many seconds does it take light to reach the Earth, if the sun is 1.5×10^{11} meters from Earth? Write the answer in scientific notation.

68. Through research, scientists found that the body of a 200 lb person consists of 3.2×10^{-5} lbs of zinc. In the bodies of 1,500 such people, how much zinc is present?

69. If we suppose that the volume of Lake Rason is approximately 2.56×10^5 gal and Lake Rason is 20 times the volume of Lake Rushy, write the volume of Lake Rushy (approximately).

70. Scientists discovered that the size of the Antarctic Ocean is 20,330,000 square feet. How big is the Arctic Ocean, if the Arctic Ocean is ¾ of the size of the Antarctic Ocean?

71. How far does light travel in water in 5.0×10^2 seconds, if the speed of light in water is 3×10^8 m/s?

72. Suppose there are 5×10^6 bacteria in every 2 liters of water. How many bacteria are there in 5 liters of water?

73. Ron has to calculate the time taken by a sound wave to travel from Earth to Venus at the speed of 4.78×10^{12} miles per year (called a light-year). The distance between Earth and Venus is 2.1×10^{19} miles.

Exponents, Radicals, & Scientific Notation

1.)	$(7)(7)$	26.)	$3^{2/6}$	51.)	0.793
2.)	$(3)(3)(3)(3)$	27.)	$17^{4/3}$	52.)	0.000742
3.)	$(5)(5)(5)$	28.)	$135^{3/2}$	53.)	971300
4.)	2	29.)	$x^{2/4}$	54.)	1256
5.)	$(x)(x)(x)(x)(x)$	30.)	$31^{2.4/5}$	55.)	0.0000083
6.)	$\dfrac{1}{(7)(7)}$	31.)	2187	56.)	4.23
7.)	1	32.)	512	57.)	301000
8.)	$(1.1)(1.1)(1.1)$	33.)	1679616	58.)	0.00000000269
9.)	$\dfrac{1}{(x)(x)(x)}$	34.)	4096	59.)	115100
10.)	$(9)(9)(9)(9)+(3)(3)$	35.)	2	60.)	73.7
11.)	5^6	36.)	128	61.)	4.08×10^{12}
12.)	20^3	37.)	50625	62.)	4.5×10^{-3}
13.)	10^8	38.)	x^2	63.)	$2.796\ x\ 10^{10}$
14.)	51^{12}	39.)	0.06	64.)	2.24×10^9
15.)	2.3^{11}	40.)	1	65.)	6.2×10^7
16.)	x^{-3}	41.)	6.05×10^9	66.)	6.5×10^{-5}
17.)	11^4	42.)	8.4179×10^7	67.)	4.5×10^{19}
18.)	7^1	43.)	7.010×10^{-9}	68.)	4.8×10^{-2}
19.)	$\left(\dfrac{3}{7}\right)^3$	44.)	$5.16\ x\ 10^{-3}$	69.)	5.12×10^6
20.)	$4^3 + 2^2$	45.)	$7.8882\ x\ 10^4$	70.)	1.52475×10^7
21.)	$\sqrt[3]{5^2}$	46.)	$3.404\ x\ 10^{-8}$	71.)	1.5×10^{11}
22.)	$\sqrt[4]{6}$	47.)	7.035×10^{-2}	72.)	$1.25\ x\ 10^7$
23.)	$\sqrt{x^5}$	48.)	2.5×10^7	73.)	1.0038×32
24.)	$\sqrt[4]{9^3}$	49.)	6.5×-1		
25.)	$\sqrt{144}$	50.)	9.78×10^0		

Section 5

Order of Operations

ORDER OF OPERATIONS

Basic Math for
Operators-in-Training
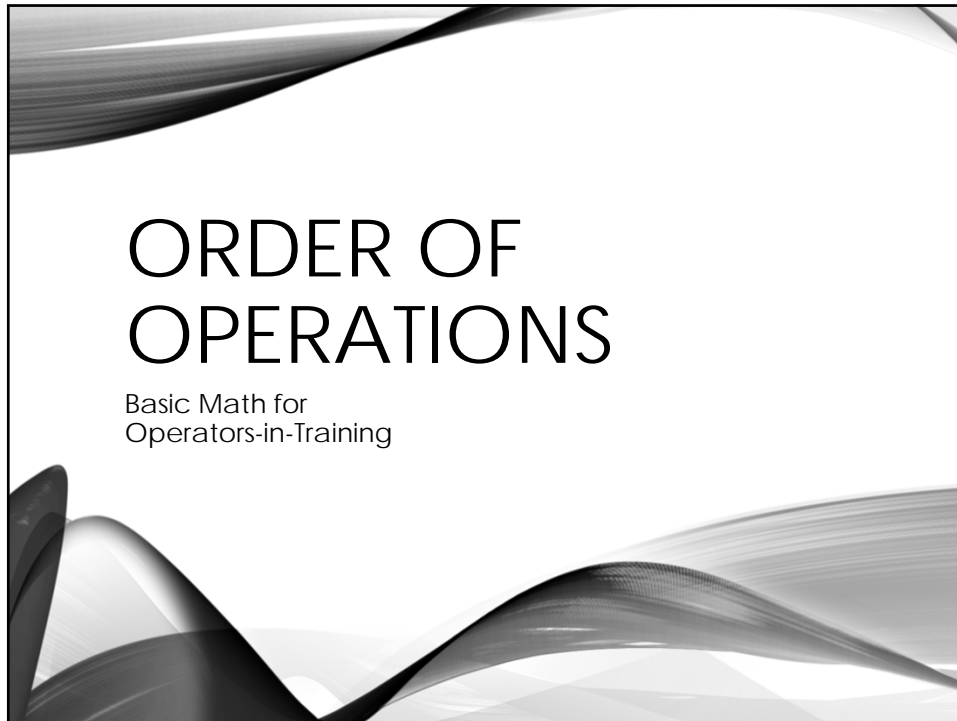
WHAT IS ORDER OF OPERATIONS?

- A set way to solve an calculation

$$8 + 16 \div 4$$

| |
| |
| |

| |
| |
| |

- Which way is the correct way?

PEMDAS

- Parenthesis
- Exponents
- Multiplication/Division
- Addition/Subtraction

$$8 + 16 \div 4$$
$$24 \div 4$$
$$6$$

$$8 + 16 \div 4$$
$$8 + 4$$
$$12$$

EXAMPLE

Parenthesis
Exponents
Multiplication/Division
Addition/Subtraction

Since Multiplication and Division are on the same "level," work left to right

Skip the subtraction because it is on the next "level"

$$(7 \times 3) \times 4 \div 2 - 5 \times 6$$

$$\underline{(7 \times 3)} \times 4 \div 2 - 5 \times 6$$

$$21 \times 4 \div 2 - 5 \times 6$$

$$\underline{21 \times 4} \div 2 - 5 \times 6$$

$$84 \div 2 - 5 \times 6$$

$$\underline{84 \div 2} - 5 \times 6$$

$$42 - 5 \times 6$$

$$42 - \underline{5 \times 6}$$

$$\underline{42 - 30}$$

$$12$$

Math Fundamentals
Order of Operations (1)
Practice Problems

1. $3(6 + 7)$

2. $5 \times 3 \times 2$

3. $72 \div 9 + 7$

4. $2 + 7 \times 5$

5. $20 \div [4 - (10 - 8)]$

6. $40 \div 4 - (5 - 3)$

7. $9 + 9 + 6 - 5$

8. $(5 + 16) \div 7 - 2$

9. $\dfrac{27}{2+3+4} + 3$

10. $2 \times 7 - \dfrac{10}{9-4}$

11. $8 \times \dfrac{15}{5} - (5 + 9)$

12. $8 + 96 \div 2$

13. $42 \div 6 - 3$

14. $42 - 15 \div 5$

15. $8 \times 6 + 5$

16. $45 + 6 \times 15$

17. $91 \div 7 - 9$

18. $848 - 2 \times 67$

19. $8 + 9 - 2 \times 3$

20. $36 \div 3 - 4 \times 3 + 24 \div 2$

21. $55 + 8 \times 2 \div 2 + 34 - 17 + 19$

22. $44 - (18 \div 9)$

23. $(73 + 4) + 14 \times 6$

24. $(9 + 5) \times (9 + 5)$

25. $[(15 + 18) - 33] \times 6$

26. $(10 + 2) + 15 \times 3$

27. $[16 - (14 \div 7)] + 82$

28. $(12 \times 7 - 82) + 10$

29. $(69 - 32) - (0 + 5)$

30. $4 \times (13 - 3) + 42$

31. $(10 - 5)(2) + [(12 + 6) \times 52]$

Answers

Order of Operations Answers		
1.) 39	11.) 27	21.) 99
2.) 30	12.) 56	22.) 42
3.) 15	13.) 4	23.) 161
4.) 37	14.) 39	24.) 196
5.) 10	15.) 53	25.) 0
6.) 8	16.) 135	26.) 57
7.) 19	17.) 4	27.) 96
8.) 1	18.) 714	28.) 12
9.) 6	19.) 11	29.) 32
10.) 12	20.) 12	30.) 82
		31.) 250

Math Fundamentals
Order of Operations
Practice Problems (2)

1.   (14 + 2) x 8 − 4

2.   4 x 3 + (3 + 6)

3.   (11 + 5) + 10 x 5

4.   (8 + 27 − 5) x 6

5.   (10 + 3) x (7 − 5)

6.   (12 + 7) x 9 + 2

7.   2 x 3 + (9 + 6)

8.   (9 + 3) + 15 x 5

9.   (10 + 20 − 6) x 6

10. (14 + 3) x (12 + 5)

11. (14 + (15 - 3)) x 7

12. 12 + ((17 + 4) + 2)

13.   (7 + (18 − 3 + 2))

14.   ((11 + 4) + 4) + 8

15.   (10 + (18 − 3)) x 7

16.   2 + ((13 + 5) + 6)

17.   ((10 − 2) x 5) − 10

18.   13 + (10 + (11 − 5))

19.   15 + (5 x (17 − 6))

20.   8 + (14 − 7 − 6))

21.   18 + (5 x (11 − 4)^2)

22.   ((14 − 2) +14 − 2)^2)

23.   14 + (5 x (4 + 3)^2)

24.   18 + ((10 + 3) + 2^2)

25.   (4^2 + (10 - 2 + 4^2))

26.   (6^2 + (20 − 5 + 3^2))

27. $18 + ((11 + 7) + 3^2)$ 29. $((18 + 2) + (20 - 4)^2)$

28. $((5 + 4)^2 \times 2) + 2^2$ 30. $((10 - 4)^2 + 6) - 4^2$

Order of Operations Answers		
1.) 124	11.) 182	21.) 263
2.) 21	12.) 35	22.) 576
3.) 66	13.) 24	23.) 259
4.) 180	14.) 27	24.) 35
5.) 26	15.) 175	25.) 40
6.) 173	16.) 26	26.) 60
7.) 21	17.) 30	27.) 45
8.) 87	18.) 29	28.) 166
9.) 144	19.) 70	29.) 276
10.) 289	20.) 9	30.) 26

Section 6

Introduction to Equations

INTRODUCTION TO EQUATIONS

IXL

EXPRESSIONS

Terminology

- A *mathematical operation* refers to calculating a value using operands and a math operator.

- The numbers used for an operation are called *operands*.

- The symbol indicating a math operation is an *operator*.
  - Examples:

+ for addition	− for subtraction
× for multiplication	÷ for division
= for equal to	

- An *inverse operation* is an operation that reverses the effect of an operation.
  - Addition & Subtraction are inverse operations.
  - Multiplication & Division are inverse operations.
  - Powers & Roots are inverse operations.

Terminology

constant   coefficient   variable

$$2 + 3x$$

first term   second term
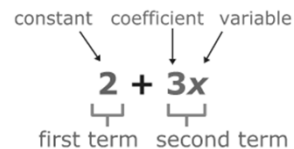
- A *variable* is a letter that represents an unknown amount.
  - Can be any letter (x, d, t, y, etc.)
  - Example: In the equation $2b+7=15$, b is the variable.

- A *coefficient* is a number multiplied by a variable. A coefficient is written immediately in front of the variable.
  - Example: In the equation, $9n - \frac{1}{2}$, 9 is the coefficient.

- A *constant* is a term that contains only a number and no variables.
  - Example: In the equation $7 - (0.6)(t)$, 7 is the constant.

- A *term* is a part of an expression or equation that is added or subtracted.
  - Example: In the expression $3x - 9 + \frac{2}{y}$, 3x, 9, $\frac{2}{y}$ are all terms.

Section 6

TDEC - Fleming Training Center

Evaluating Expressions

- An *expression* is a mathematical phrase that contains numbers, variables, or both.
  - Expressions never have an equal sign.

$$10 + 5 \qquad 370 - x$$

- Example:

Evaluate the expression $\frac{1}{2}(b)(h)$, if b=7cm and h=4 cm.

$$\left(\tfrac{1}{2}\right)(b)(h)$$

$$\left(\tfrac{1}{2}\right)(7 \text{ cm})(4 \text{ cm})$$

$$14 \text{ cm}^2$$

Evaluating Expressions

- When you evaluate an expression, you are finding the value of the expression.
  - If the expression has a variable, you will need to replace the variable with a given value. This is called *substitution*.

- To evaluate a variable expression:
  1. Substitute the value of the variable into the expression.
  2. Follow the order of operations.

- Example: Evaluate $x + 8$ when $x = 4$.
  1. Replace x with 4.

$$x + 8$$
$$4 + 8$$

  2. Then, add.

$$4 + 8 = 12$$

So, $x + 8 = 12$ when $x = 4$.

Introduction to Equations

Operation	Keywords	Example	Algebraic Expression
Addition	more than	6 more than x	$x + 6$
	the sum of	the sum of n and 3	n + 3
	increased by	k increased by 10	k + 10
Subtraction	less than	8 less than a	a - 8
	the difference of	the difference of 16 and m	16 - m
	decreased by	r decreased by 12	r - 12
Multiplication	the product of	the product of 5 and g	(5)(g)
	twice	twice y	(2)(y)
	double	double t	(2)(t)
	triple	triple v	(3)(v)
Division	the quotient of	the quotient of z and 8	z ÷ 8
	split into	d split into 5 groups	d ÷ 5
	half	split p in half	$\dfrac{p}{2}$
	third	a third of c	$\dfrac{c}{3}$

EQUATIONS

Fleming Training Center

TN Department of Environment & Conservation

Introduction to Equations

- An *equation* is a mathematical sentence that says two expressions are equal.
  - Equations always have an equal sign.
  - Equations are "balanced."
    - The quantity on one side of the equal sign is equivalent to the quantity on the other side of the equal sign.
    - To maintain that balance, we whatever we do to one side of the equation, we must do the same to the other side.

Evaluating Equations

- An *equation* is a mathematical sentence that says two expressions are equal. Some equations have a variable.

- A *variable* is a letter that represents an unknown amount.

- To find the solution of an equation with a variable, you must find a value for that variable that makes the equation true (balanced).

- You can often find the solution by getting the variable alone on one side of the equation.
  - Use inverse operations to accomplish.

General Rules of Thumb

- Get variable in numerator.

- Get variable positive.

- Get variable on one side of the equal sign alone.

- Whatever is done to one side of the equation must be done to the other side to maintain the equation's balance.

- When solving for a variable, follow the Order of Operations BACKWARDS.

Evaluating One-Step Equations with Addition & Subtraction

- To solve a equation that contains addition or subtraction:
  1. Get the variable alone by applying the inverse operation to both sides of the equation.
  2. Simplify.

- Example: Solve $k - 19 = 51$.

  - Since 19 is on the same side of the equal sign as k, we need to perform the inverse operation to eliminate 19.
  - 19 is subtracting from k, so to eliminate it we will do the inverse operation: subtraction.
    - ❖ To maintain equation balance, whatever we do to one side of the equal sign, we must do the other.

Evaluating One-Step Equations with Addition & Subtraction

- Example (cont'd):   Solve $k - 19 = 51$.

$$k - 19 = 51$$
$$+ 19 \ + 19$$

$$k - 19 + 19 = 51 + 19$$

$$k - 0 = 51 + 19$$

$$k = 70$$

Whatever you do to one side of the equation, you must do the other side, as well.

- You can check your solution by replacing the variable in the original equation with the value we just found.

$$70 - 19 = 51$$

If $k = 70$, then $k - 19 = 51$.

Evaluating One-Step Equations with Multiplication & Division

- To solve a equation that contains multiplication or division:
  1. Get the variable alone by applying the inverse operation to both sides of the equation.
  2. Simplify.

- Example: Solve $(m)(21) = 252$.

- Since *21* is on the same side of the equal sign as *m*, we need to perform the inverse operation to eliminate *21*.
- *21* is multiplying *m*, so to eliminate it we will do the inverse operation: division.
  - ❖ To maintain equation balance, whatever we do to one side of the equal sign, we must do the other.

Evaluating One-Step Equations with Multiplication & Division

- Example: Solve $(m)(21) = 252$.

$$\frac{(m)(21)}{21} = \frac{252}{21}$$

Whatever you do to one side of the equation, you must do the other side, as well.

$$\frac{(m)(\cancel{21})}{\cancel{21}} = \frac{252}{21}$$

$$m = \frac{252}{21}$$

$$m = 12$$

- Check your answer by plugging value for m into original equation.

$$(12)(21) = 252$$

Evaluating Two-Step Equations

- Sometimes you will have to complete more than one inverse operation to solve an equation with a variable.
- When solving an equation with a variable, follow order of operations backwards:

SADMEP

- Example: Solve $(7)(x) - 3 = 25$.
  - There are two operations occurring in this equation:

Multiplication
Subtraction

  - Subtraction will be the first operation to eliminate.
  - ❖ To maintain equation balance, whatever we do to one side of the equal sign, we must do the other.

$$(7)(x) - 3 = 25$$
$$\underline{+3 \quad +3}$$
$$(7)(x) - 0 = 28$$

Evaluating Two-Step Equations

- Example (cont'd):

$$(7)(x) = 28$$

  - Now the 7 is multiplying x.
  - To eliminate the 7, we must perform the inverse operation: division.

$$\frac{(7)(x)}{7} = \frac{28}{7}$$

Whatever you do to one side of the equation, you must do the other side, as well.

  - Eliminate any common terms in the numerator and denominator.

$$x = \frac{28}{7}$$

  - Simplify.

$$x = 4$$

Math Fundamentals
Expressions and Equations
Practice Problems

Write an expression for each word problem.

1. Seven more than five times a number.

2. Two more than nine times a number.

3. Nine more than eight times a number.

4. Five more than six times a number.

5. A number increased by 10.

6. Thrice a number, then increased by fourteen

7. Twenty-one increased by four times a number.

8. Eleven diminished by seven times a number.

9. Ten increased by three times a number.

10. Forty decreased by twelve times a number.

Simplify the following expressions. Answer will contain a variable.

11. $g + 10g =$ _____

12. $4b + 7b =$ _____

13. $15h - 12h =$ _____

14. $5c - 2c + 7c =$ _____

15. $3 \times (8n + 4) =$ _____

16. $7.2m \times 2.4 =$ _____

17. $\left(\dfrac{248}{62}\right) d =$ _____

18. $\left(\dfrac{51}{119}\right) y \times 17 =$ _____

19. $\dfrac{315}{126m} =$ _____

20. $(7)(f) - 16 =$ _____

21. Joe Smore weighs 105 pounds. Every year his weight increases by 3 pounds. <u>Write an expression</u> to show what Joe's weight will be in **X** years' time.

    a. 105+3x

22. The area of a sea horse is 108 cm. Every day it grows by 2%. <u>Write and solve an expression</u> that represents the area of a sea horse after 3.5 days.

    a. 108 + (108 x 0.02 x 3.5)

23. Rachel fills 13 water bottles per hour. Alyssa fills 15 bottle of water per hour. This week Alyssa filled an additional 5 bottles of water. <u>Write an expression </u>that represents the weekly water of bottle filled.

   R = the number of hours that Rachel filled bottles this week.

   A = the number of hours Alyssa filled the bottles this week.
   a. 13R + 15A

24. Smith has **P** small balloons and 20 big balloons. <u>Write an expression</u> that shows how many balloons Smith has.
   a. 20 + p

25. Johnson has 19 bowls of ice cream. He gives away **D** ice cream bowls. <u>Write an expression</u> that shows the number of ice cream bowls that are left.
   a. 19-d

26. Kim bought a purse (**B**). Then she went to the market and bought 16 more purses. <u>Write an expression</u> that shows how many purses Kim has now.
   a. B + 16

27. Moore has **V** small chairs and 22 big chairs. <u>Write an expression</u> that shows how many chairs Moore has.
   a. V + 22

28. Nancy has 62 more erasers than Barbara. Barbara has **F** erasers. <u>Write an expression</u> that shows how many erasers Nancy has.
   a. F + 62

29. David has **P** leaves. He planted 22 more leaves. <u>Write an expression</u> that shows how many leaves he has now.
    a. P + 22

30. Arial ate 90 out of E candies. <u>Write an expression</u> that shows how many candies he has left.
    a. 90-E

31. Kerry has 77 dolls. Perry has **D** more dolls than Kerry. <u>Write an expression</u> that shows how many dolls Perry has.
    a. 77+D

32. Clark has **P** apples. Paul has 11 fewer apples. <u>Write an expression</u> that shows how many apples Paul has.
    a. P-11

Solve for the variable in the following equations. Round answers to second decimal point.

<u>Addition</u>

33. $3 + g = 10$

34. $x + 2 = 3$

35. $x + 15 = 19 + 22$

36. $7 + 10 + x + 7 + 9 = 41$

37. $x + 93 = 165$

Subtraction

38. $3 = k - 2$

39. $x - 2 = 9$

40. $x - 93 = 65$

41. $9.5 - x = 8.7$

42. $115 = x - 7.5$

Multiplication

43. $10 = (2)(w)$

44. $(5)(m) = 10$

45. $48 = (6)(m)$

46. $8.1 = (3)(x)(1.5)$

47. $(0.785)(0.33)(0.33)(x) = 0.49$

Division

48. $12 = \dfrac{t}{8}$

49. $\dfrac{2}{e} = 6$

50. $\dfrac{100}{x} = 50$

51. $56.5 = \dfrac{3800}{(x)(8.34)}$

52. $114 = \dfrac{(230)(1.15)(8.34)}{(0.785)(70)(70)(x)}$

Assorted

53. $\dfrac{(8.2 - h)}{7.9} = 0.86$

54. $\dfrac{(n - 4.5)}{2.9} = 7.5$

55. $4.2(d + 4.4) = 19.1$

56. $2.8 + \left(\dfrac{1}{9}\right)(c) = 5.4$

57. $\left(\dfrac{1}{3}\right)(y) + 5.6 = 4.2 + 1.4$

58. $(5.3)(a) + 6.4 = 9.7$

59. $\dfrac{(8.2+x)}{6.8} = 7.5$

60. $6.6 + (8.1)(w) = 20.1$

61. $8.1 = (1.5)(x)(3)$

62. $56.5 = \dfrac{3800}{(x)(8.34)}$

Write an equation for each word problem. Do not solve for the variable.

63. Four less than a number **X** is sixty-four.

64. Nine less than a number is twenty.

65. The sum of eight and a number is five less than seventy.

66.   Two more than a number is Fifty.

67.   The sum of **N** and 5 is five less than fifteen.

68.   A number minus 10 is thirty.

69.   The sum of a number and five is thirteen.

70.   The sum of **A** and eight is twenty.

71.   A number three times is twenty-seven.

72.   Twice a number is two more than six.

<u>Solve</u> the following word problems. Round answers to second decimal point.

73.   Bill had some burgers. He ate two burgers and then he divided the remaining burgers amongst his friends. Each of his 5 friends got 2 burgers. <u>Write and solve an equation</u> to find the number of burgers Bill started with.

74. Five hundred (500) students went on a field trip. Ten (10) students travelled in each car. <u>Write and solve an equation</u> to find the number of cars (**X**) used for the field trip.

75. John bought ten books all at the same price. The total cost of the books was $242. He gave two books to his friend. <u>Write and solve an equation</u> to find the value of each book (**X**) that John gave to his friend.

76. Jack's Corner Shoppe had a huge back to school special. For $1,000 you get seven (7) desk chairs, **X** tables, and two laptops. <u>Write and solve an equation</u> to find the number of desks sold if Jack made $4,000.

77. Jorge Handy is one of the Football league star players. He is also somewhat of a statistical phenomenon. For every point he scores, he has 3 receiving yards, and 5 rushing yards. <u>Write and solve an equation</u> that would allow you to determine the total number of yards Jorge scored in the game, if he had 21 points.

78. Nancy sold half of her comic books and then bought 8 more. She now has 13. How many did she begin with?

79. Ron works 5 hours for a total of $300. He deposits all of his money is his account. He is trying to save $500 for a new LED TV. <u>Write and solve an equation</u> to find the number of hours Ron will need to work to earn $500 for his new TV.

80. Sam bought 5 new baseball trading cards to add to his collection. The next day his dog ate half of his collection. There are now only 32 cards left. How many cards did Sam start with ?

81. Samuel has $1,000 in his saving account. Samuel buys a new front door for his home for $700. After the purchase, he wants to withdraw $50 a week from his account for personal expenses. <u>Write and solve an equation</u> to find how many weeks Samuel can withdraw money before running out.

82. James scored 300 runs in cricket. He requires 900 runs to qualify for league MVP contention. He scores 50 runs during the average game. <u>Write an equation</u> to find the number of games James will need to play to score the remaining runs he needs.

83. Joshua sells 13 toys per hour. Sarah sells 6 toys per hour. Together this week they made 215 toys. <u>Write and solve an equation</u> that represents the number of hours (x) Joshua and Sarah worked.

84. Tom spent half of his allowance going to the movies. He washed the family car and earned six dollars. What is his weekly allowance if he ended with fourteen dollars?

85. Victoria has a dog. The dog's tail is 2 inches long. Every year his tail grows by 5%. <u>Write and solve an equation</u> to show how many inches the tail will grow in 6 years.
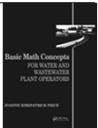
86. Anthony purchases two bags. The price of all bags is $5.20. Anthony purchases one school bag and one handbag. <u>Write and solve an equation</u> that represents the total cost of the bag if s represents the number of school bags and h represents the number of hand bags.

87. David made 30 cups of tea per hour. Andrew made 35 cups of coffee per hour. This week David made an additional 10 cups of tea. If 1167 cups of drinks were made, <u>write and solve and equation</u> that represents the total number of cups of tea and coffee made by David and Andrew.

88. Sam bought a soft drink for 4 dollars and 5 candy bars. He spent a total of 29 dollars. How much did each candy bar cost?

89. On Monday, 457 students went on a trip to the zoo. All 9 buses were filled, and 7 students had to travel in cars. How many students were in each bus ?

90. The sum of three consecutive even numbers is one hundred forty - four. What is the smallest of the three numbers ?

Expressions and Equations Answers			
1.) 7 + 5x	24.) 20 + P	46.) 1.8 = x	69.) x + 5 = 13
2.) 2x + 9	25.) 19 − D	47.) x = 5.73	70.) A + 8 = 20
3.) 9 + 8x	26.) B + 16	48.) 96 = t	71.) D x 3 = 27
4.) 5 + 6x	27.) V + 22	49.) 0.33 = e	72.) 2B = 2 x 6
5.) x+10	28.) F + 62	50.) 2 = x	73.) 12
6.) 3x + 14	29.) P + 22	51.) x = 8.06	74.) 50
7.) 21 + 4x	30.) 90 − E	52.) x = 0.0050	75.) 24.20
8.) 11 − 7x	31.) 77 + D	53.) 1.41 = h	76.) 4
9.) 10 + 3x	32.) P − 11	54.) n = 66.75	77.) 168
10.) 40 − 12x	33.) g = 7	55.) d = 0.14	78.) 10
11.) 11g	34.) x =1	56.) c = 23.4	79.) 10
12.) 11b	35.) x = 26	57.) y = 0	80.) 59
13.) 3h	36.) x = 8	58.) a = 0.62	81.) 6
14.) 10c	37.) x = 72	59.) x = 37	82.) 12
15.) 24n + 12	38.) 5 = k	60.) w = 1.67	83.) 11
16.) 71.28m	39.) x = 11	61.) 43.75 = x	84.) 16
17.) 4d	40.) x = 160	62.) x = 11.85	85.) 0.6
18.) 7.29y	41.) 0.8 = x	63.) x − 4 = 64	86.) 36.40
19.) $\dfrac{2.5}{m}$	42.) 122.5 = x	64.) x − 9 = 20	87.) 18
20.) 7f - 16	43.) 5 = w	65.) 8 + x = 70 − 5	88.) 5
21.) 105 + 3x	44.) m = 2	66.) 2x = 50	89.) 50
22.) 108 + (108 x 0.02 x 3.5)		67.) N + 5 = 15 − 5	90.) 48
23.) 13R + 15A	45.) 4 = m	68.) x − 10 = 30	

Section 7

Ratios & Proportions

RATIOS AND PROPORTIONS

Chapter 7
Determining Proportions
Solving a Proportion
Setting up a Proportion

Ratios & Proportions

- A *ratio* is the established relationship between two numbers.
  - Example:　　　12 inches is 1 foot
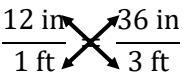
    12 inches: 1 foot

    12 inches = 1 foot

    $$\frac{12 \text{ inches}}{1 \text{ foot}}$$

- A *proportion* is an equation in which two ratios are set equal to each other.
  - Example:　　　12 inches is 1 foot; 36 inches is 3 foot

    12 inches : 1 foot = 36 inches : 3 foot

    $$\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{36 \text{ inches}}{3 \text{ foot}}$$

Determining Proportions

- Cross Multiplication
  - Used if proportion is set up using fractions.

$$\frac{12 \text{ in}}{1 \text{ ft}} \diagdown \diagup \frac{36 \text{ in}}{3 \text{ ft}}$$

$$(12 \text{ inches})(3 \text{ ft}) = (1\text{ft})(36\text{in})$$

$$36 = 36$$

- Division Method
  - Divide each ratio and compare answers.

$$\frac{12 \text{ in}}{1 \text{ ft}} = \frac{36 \text{ in}}{3 \text{ ft}}$$
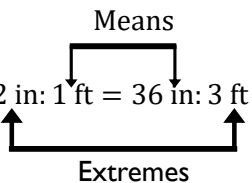
$$12 \div 1 = 12$$
$$36 \div 3 = 12$$

Determining Proportions

- Means and Extremes Method
  - Used if the proportion is written using colons
  - Product of the means will be equal to the product of the extremes

Means

12 in: 1 ft = 36 in: 3 ft

Extremes

$$(1\text{ft})(36\text{in}) = (12\text{in})(3\text{ft})$$

$$36 = 36$$

Solving a Proportion

- Solving for Unknown Method
  - Four (4) terms in every proportion
  - In a proportion problem, 3 terms are known and 1 is unknown
  - Becomes a solving for unknown problem

- Example:

$$\frac{26}{190} = \frac{x}{4750}$$

$$\frac{4750}{1} \times 0.1368 = \frac{x}{4750} \times \frac{4750}{1}$$

$$(4750)(0.1368) = x$$

$$649.8 = x$$

$$650 \approx x$$

Solving a Proportion

- Cross Multiplication Method
  - Used often when *unknown* is in the denominator
  - Example:

$$\frac{26}{190} \diagdown \frac{x}{4750}$$

$$(26)(4{,}750) = (190)(x)$$

$$\frac{123{,}500}{190} = \frac{(190)(x)}{190}$$

Whatever you do to one side of the equation, you must do the other side, as well.

$$\frac{123{,}500}{190} = x$$

$$650 = x$$

Solving a Proportion

- Means & Extremes Method
  - Example: $\qquad 26 : 190 = x : 4750$
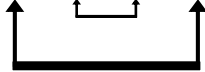
$$(26)(4750) = (190)(x)$$

$$650 = x$$

- Equivalent Fractions Method
  - Find value that makes fractions equivalent.
  - Example: $\qquad 1 : 4 = x : 20$

$$\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$$

$$x = 5$$

Setting up a Proportion Problem

- To set up and solve direct proportions:
  1. Write the two fractions ensuring that unit setup matches on both sides of equal sign.
  2. Fill in given values for both fractions.
     - One value will be x
  3. Solve for the unknown value.

- To set up and solve indirect proportions:
  1. Group like units.
  2. Place the smaller numbers in the numerators and the larger numbers in the denominators.
  3. Solve for the unknown value.

Direct vs Inverse Proportions

- In *direct proportions* an increase in one variable cause the other variable to increases too and vice versa.

↑ ↑   ↓ ↓

As one number   As one number
increases, the   decreases, the
other number   other number will
will increase, too.   decrease, too.

- In *inverse proportions* an increase in one variable causes a decrease in the other variable, and vice versa.

↑ ↓   ↓ ↑

As one number   As one number
increases, the   decreases, the
other number   other number
will decrease.   will increase.

Solving a Direct Proportion Problem

- Most proportions in the water industry are made up of direct ratios.

  feet : inches ft^3 : gal   gallons: square feet

         chlorine concentration : contact time

- To solve a direct proportion:

1. Determine the two ratios to be compared.

2. Rewrite the two ratios as a fractional proportion.
   - Take care to make the units match on the left side and right side of equal sign.

3. Fill in numbers that accompany units of measure.

4. Solve for the unknown value.

Direct Proportion Problem - Example

•If one (1) pound of chlorine is dissolved in 25 gallons of water. How many pounds of chlorine would be dissolved in 63 gallons of water?

1. 1 pound ∶ 25 gallons   **?** pounds ∶ 63 gallons

2. $\dfrac{\text{pounds}}{\text{gallons}} = \dfrac{\text{pounds}}{\text{gallons}}$

3. $\dfrac{1 \text{ pound}}{25 \text{ gallons}} = \dfrac{X \text{ pounds}}{63 \text{ gallons}}$

Direct Proportion Problem - Example

4. At this point, you can choose how you solve the proportion:
   a) Solve for X
   b) Cross multiplication

Solve for X

$$\frac{63 \text{ gallons}}{1} \times \frac{1 \text{ pound}}{25 \text{ gallons}} = \frac{X \text{ pounds}}{63 \text{ gallons}} \times \frac{63 \text{ gallons}}{1}$$

$$\frac{(63 \text{ gallons})(1 \text{ pound})}{25 \text{ gallons}} = X \text{ pounds}$$

$$2.52 \text{ pounds} = X \text{ pounds}$$

Cross Multiplication

$$\frac{1 \text{ pound}}{25 \text{ gallons}} = \frac{X \text{ pounds}}{63 \text{ gallons}}$$

$$(63 \text{ gallons})(1 \text{ pound}) = (25 \text{ gallons})(X \text{ pounds})$$

$$2.52 \text{ pounds} = X \text{ pounds}$$

Indirect Proportion Problem - Example

- To set up and solve an indirect proportion:
  1. Group like units.
  2. Place the smaller numbers in the numerators and the larger numbers in the denominator.
  3. Solve for the unknown value.

- Example: If it takes 3 men 60 hours to complete a job, how many hours will it take 5 men to complete the same job?

Step 1 & 2. $\dfrac{3 \text{ men}}{5 \text{ men}} = \dfrac{\text{X hours}}{60 \text{ hours}}$

Step 3. Solve the proportion with your preferred method.

$$36 \text{ hours} = \text{X hours}$$

Math Fundamentals
Proportions (1)
Practice Problems

Use cross-multiplication to solve the following proportions.

1. $\dfrac{x}{6} = \dfrac{15}{18}$

2. $\dfrac{16}{24} = \dfrac{x}{3}$

3. $\dfrac{5}{3} = \dfrac{15}{x}$

4. $\dfrac{x}{6} = \dfrac{2}{12}$

5. $\dfrac{w}{5} = \dfrac{16}{20}$

6. $\dfrac{2}{1} = \dfrac{8}{x}$

7. $\dfrac{5}{y} = \dfrac{10}{16}$

8. 25 : d = 10 : 2

9. 15 : 3 = h : 4

10. m : 30 = 8 :12

Setup and solve a proportion to solve the following word problems.

11. The ratio of tomatoes to red apples is 2:5. If there are 20 tomatoes, how many red apples are there?

12. The ratio of berries to oranges is 10:1. If there are 25 oranges, how many berries are there?

13. The ratio of potatoes to turnips is 1:1. If there are 473 potatoes, how many turnips are there?

14. The ratio of pears to green apples is 1:3. If there are 150 green apples, how many pears are there?

15. The ratio of bananas to melons is 30:1 If there are 300 bananas, how many melons are there?

16. The ratio of blueberries to strawberries is 1:7. If there are 210 strawberries, how many blueberries are there.

17. The ratio of boys to girls is 3:2. If there are 243 boys, how many girls are there?

18. The ratio of students with blond hair to students with brown or black hair is 1:2. If there are 212 students with brown or black hair, how many students have blond hair?

19. 60 students are in grade 1, but only 30 students are in kindergarten. What is the ratio of grade 1 students to kindergarten students? Give answer in reduced form.

20. The ratio of students who made the honor roll to the total number of students is 1:50. If there are 500 students in total, how many made the honor roll?

21. 12 students play on the school's basketball team and 36 students play on the football team. What is the ratio of basketball players to football players?
    a.

22. The ratio of students getting an "A" in math to students getting a "B" is 1:2. If 22 students received an "A", how many received a "B"?
    a.

23. A car travels 120 miles in 3 hours (with a constant speed). How far will it take to travel 200 miles?

24. If 50 apples cost $25, how much would 75 apples cost?

25. It takes Mike 18 minutes to finish reading 4 pages of a book. How long does it take for him to finish reading 30 pages?

26. Nathan packs 25 boxes in 2 hours. How many boxes can he pack in his 8-hour shift?

27. 13 candy bars weigh 26 ounces. What is the weight of 35 candy bars?

28. A machine can produce 6 yards of fabric in 2 minutes. How much fabric can the machine produce in 1 hour?

29. A bus travels 350 km in 4 hours (with a constant speed). How far can it travel in 7 hours (with the same speed)?

30. If two water towers can hold 905 gallons of water, how much water can 12 water towers can hold?

31. If it takes four moving trucks to move 48 boxes, how many trucks are needed to move 840 boxes?

32. A factory can produce 30 TV in 9 days. How long does it take to finish 400 TVs?

33. 500 lemons were packed in 8 boxes. How many lemons can be packed in 20 boxes?

34. A pack of four cans of coffee cost $10.90. How much would 18 cans of coffee cost?

35. 24 loaves of bread cost $48. How much does 10 loaves cost?

36. A chef made 30 donuts in 60 minutes. How long would it take him to make 90 donuts?

37. Four big water bottles can hold 8 gallons of water. How much water can ten big water bottles hold?

38. It took Nora 10 hours to walk a 30-mile trail. How long did it take her to walk 9 miles at the same speed?

39. The total weight of 15 boxes is 45 pounds. How much would 40 boxes weigh?

40. A pack of six cans of coffee cost $12. How much would 15 cans of coffee cost?

Math Fundamentals
Proportions Practice Problems

1. There are a total of 63 bikes. If the ratio of blue bikes to black bikes is 4 to 5, how many of the bikes are blue?

2. Paul can walk 15 steps in 5 minutes. How long does it take Paul to walk 75 steps at the same speed?

3. Candy is at the balloon shop and sees that 10 balloons cost $0.15. He needs 50 balloons to decorate his room. How much will 50 balloons cost?

4. It takes 8 people to pull a 16 ton truck. How many people would it take to pull a 60 ton truck?

5. If a globe rotates through 150 degrees in 5 seconds, how many degrees does it turn in 30 seconds?

6. If the ratio of white pens to green pens is 2 to 8 and there are a total of 30 pens, how many pens are white?

7. Giles is searching for a sock and discovers that he has 10 socks for every 5 pairs of shoes. If he has 20 socks, how many pairs of shoes does he have?

8. The sum of two numbers is 150. The ratio of the same two numbers is 3:2. Find the bigger number.

9. If the ratio of pink flower to blue flower is 3 to 6 and there are total 72 flowers, how many of them are pink?

10. There are a total of 18 chairs. If the ratio of the red chairs to brown chairs is 2 to 4, how many of them are red?

11. Rock can read 10 books in 30 minutes. How long does it take Rock to read 15 books, if the speed is consistent?

12. Ricky is at the bakery shop when he sees that 8 pastries cost $160. He needs 16 pastries. How much will 16 pastries cost?

13. It takes 10 people to pull a 50 ton bus. How many people would it take to pull a 100 ton bus?

14.   If a ball rotates 110 degrees in 8 seconds, how many degrees does it rotate in 32 seconds?

15.   If the ratio of red roses to yellow roses is 4 to 6 and there are a total of 50 roses, how many of them are yellow?

16.   Freddy is searching for a shirt and discovers that he has 12 shirts for every 6 pair of jeans. If he has 18 shirts, how many pairs of jeans does he have?

17.   The sum of two numbers is 200 and the two numbers are in a ratio of 4:6. Find the larger number.

18.   If the ratio of the purple flowers to black flowers is 4 to 8 and there are a total of 108 flowers, how many of the flowers are purple?

19.   There is a total of 25 bicycles. If the ratio of gray bicycles to black bicycles is 6 to 9, how many of them are black?

20.   Ritz can eat 8 apples in 15 minutes. How long does it take Ritz to eat 16 apples at the same rate?

21.   Tom is at McDonalds and he sees that 2 burgers cost $40. He needs 12 burgers. How much will 12 burgers cost?

22.   It takes 12 people to pull 30 tons of goods. How many people would it take to pull 60 tons of goods?

23.   If a tire rotates through 250 degrees in 15 seconds, how many degrees does it rotate in 45 seconds?

24.   If the ratio of purple bikes to red bikes is 8 to 12 and there are a total of 100 bikes, how many of them are purple?

25.   Andrew is searching for a cup and discovers that he has 20 plates for every 5 pairs of cups. If he has 40 plates, how many pairs of cups does he have?

26.   The sum of two numbers is 80. The ratio of those two numbers is 3:5. Find the larger number.

27.   If the ratio of red hair bands to green hair bands is 5 to 9 with a total of 70 hair bands, how many of them are green?

28.   There are a total of 100 balloons. If the ratio of yellow balloons to blue balloons is 8 to 12, how many of them are yellow?

29.   Furry can eat 10 mangoes in 5 minutes. How long does it take Furry to eat 18 mangoes at the same speed?

30.   Harry is at the Pizza Hut and he sees that 5 pizzas cost $300. He needs 15 pizzas. How much do 15 pizzas cost?

31.   It takes 24 people to pull a 50 ton iron rod. How many people would it take to pull a 150 ton iron rod?

32.   If a coin rotates through 160 degrees in 6 seconds, how many degrees does it rotate in 60 seconds?

33.   If the ratio of blue shirts to green shirts is 5 to 12 with a total of 340 shirts, how many of the shirts are blue?

34.   Rex is searching for bread and discovers that he has 40 buns for every 10 cubes of cheese. If he has 80 buns, how many cubes of cheese does he have?

35.   The sum of two numbers is 500 with a ratio of 15:10. Find the larger number.

36.   If the ratio of silver nail paints to golden nail paints is 4 to 2 and there are total 60 nail paints, how many of them are golden?

37.   There are a total of 18 frocks. If the ratio of purple frocks to white frocks is 2 to 4, how many of them are white?

38.	Johnny can eat 5 chocolates in 2 minutes. How long does it take Johnny to eat 10 chocolates at the same rate?

39.	Gerry is at Dominos and sees that 2 choco-lava cakes cost $50. He needs 6 choco-lava cakes. How much will those 6 choco-lava cakes cost?

40.	It takes 16 people to pull a 30 ton cable wire. How many people would it take to pull a 60 ton cable wire?

41.	If a marble rotates through 180 degrees in 9 seconds, how many degrees does it rotate in 18 seconds?

42.	If the ratio of black jeans to blue jeans is 6 to 18 with a total of 240 jeans, how many of them are blue?

43.	Rex is searching for shoes and discovers that he has 25 pairs of shoes for every 15 pairs of socks. If he has 50 pairs of shoes, how many socks does he have?

44.   The sum of two numbers is 300 and the two numbers have
        a ratio of 20:10. Find the larger number.

45.   If the ratio of red roses to pink roses is 5 to 4 and there are
        a total of 45 roses, how many of them are red?

Proportions Practice Problems (2) Answers					
1)	28	16)	9	31)	72
2)	25	17)	120	32)	1600
3)	0.75	18)	36	33)	100
4)	30	19)	15	34)	20
5)	900	20)	30	35)	300
6)	6	21)	240	36)	20
7)	10	22)	24	37)	12
8)	90	23)	750	38)	4
9)	24	24)	40	39)	150
10)	6	25)	10	40)	32
11)	45	26)	50	41)	360
12)	320	27)	45	42)	180
13)	20	28)	40	43)	30
14)	440	29)	9	44)	200
15)	30	30)	900	45)	25